LAW OF PRODUCTION AND LAWS OF ALGEBRA: THE HUMBUG PRODUCTION FUNCTION: A COMMENT

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Mr. Shaikh's article is based on misconception pure and simple. The factor-share device of my 1957 article is in no sense a *test* of aggregate production functions or marginal productivity or of anything else. It merely shows how one goes about interpreting given time series if one starts by *assuming* that they were generated from a production function and that the competitive marginal-product relations apply. Therefore, it is not only not surprising but it is exactly the point that if the observed factor shares were exactly constant the method would yield an exact Cobb-Douglas and tuck everything else into the shift factor. That is what one would *want* such a method to do.

The point is even simpler than Mr. Shaikh makes it out to be. It is hardly a deep thought that for any time series g_q, g_k and s (where g_x stands for the rate of growth of x) one can always write an *exact* relation of the form $g_q = sg_k + g_A$. It is only necessary to define the time series g_A to be $g_q - sg_k$. Once that is done, it is hardly surprising that $g_q - g_A$ should equal sg_k. The only empirical questions here are, first, whether s is related to k is any systematic way in the data and, second, whether the calculated g_A satisfies any natural a priori restrictions. Mr. Shaikh ignores the first by discussing only the case where s is a constant or near-constant time series and begs the second in a sentence. They are, nevertheless, the important questions.

The cute HUMBUG numerical example tends to bowl you over at first, but when you think about it for a minute it turns out to be quite straightforward in terms of what I have just said. The made-up data tell a story, clearer in the table than in the diagram. Output per worker is essentially constant in time. There are some fluctuations but they are relatively small, with a coefficient of variation about 1/7. The fact that the fluctuations are made to spell HUMBUG is either distraction or humbug. The series for capital per worker is essentially a linear function of time. The wage share has small fluctuations which appear not to be related to capital per worker. If you ask any systematic method or any educated mind to interpret those data *using a production function and the marginal productivity relations*, the answer will be that they are exactly what would be produced by technical regress with a production function that must be very close to Cobb-Douglas.

All this has literally nothing to do with the question whether the empirical basis of aggregate productions is strong or weak. When someone claims that aggregate production functions work, he means (a) that they give a good fit to input-output data *without* the intervention of data deriving from factor shares; and (b) that the function so fitted has partial derivatives that closely mimic observed factor prices.[1] Mr. Shaikh omits to mention that this is the procedure followed by Professor Fisher in his simulation exercises (and in some follow-up experiments that Fisher and I are doing together). Thus, the last paragraph of Mr. Shaikh's Introduction is simply nonsense.

If Mr. Shaikh were really interested in understanding why aggregate production functions work (if, in fact, they do), he would have tried such an experiment on his humbug data. Instead of speculating as to why he did not, I have done it for him (with the help of Mr. Samuel L. Myers). If I regress log q on log k and time, I get

$$\log q = \underset{(0.52072)}{-0.14090} + \underset{(0.01246)}{0.00532t} - \underset{(0.76098)}{0.33071 \log k}$$

where the standard errors are given under each estimated coefficient. The squared multiple correlation is 0.0052. No coefficient is as much as half its standard error, and the point estimate of the coefficient of log k is negative. If this were the typical outcome with real data, we would not now be having this discussion. The humbug seems to be on the other foot.

Received for publication March 23, 1973. Accepted for publication March 28, 1973.

[1] Sometimes the roles are reversed and it is claimed that production functions estimated from factor-prices give a good fit to input-output data. It should be emphasized that technical change is always represented by a smooth function of time (or something else) and part of the test is whether the residuals are well-behaved.