

## The Review of Economic Studies Ltd.

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A Theory of the Business Cycle

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Source: *The Review of Economic Studies*, Vol. 4, No. 2 (Feb., 1937), pp. 77-97

Published by: [The Review of Economic Studies Ltd.](#)

Stable URL: <http://www.jstor.org/stable/2967606>

Accessed: 12/11/2010 11:56

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# A Theory of the Business Cycle

## INTRODUCTION

1. THIS paper, in which I attempt to give an analysis of investment processes, is closely allied to the Keynesian theory. The latter can be divided into two parts: (1) the determination of short-period equilibrium with a given capital equipment and with a given rate of investment; (2) the determination of the rate of investment. In the section "Short-period Equilibrium" I give a representation of the first part of the Keynesian theory, arriving at its chief theorems in a slightly different way. In the following three sections I deal with the determination of the rate of investment and there the results are fundamentally different from those of the Keynesian theory. These divergences are due to the important rôle played in my arguments by the time-lag between investment decisions and investment production, and also to a different treatment of the question of the inducement to invest. In the last section I show that the investment processes necessarily create a business cycle.

2. I assume in the whole paper that the workers do not save (or dis-save). For the savings of workers certainly do not play an important part in the economic process, while to take it into consideration can often obscure some essential features of the capitalist economy. Therefore, it seems to me preferable to deal here with a system in which only capitalists (entrepreneurs and rentiers) save—exactly as is usually admitted in the assumption of a closed economy as being justifiable in a first approach. (I assume, also, in the whole paper a closed economic system.)

The second simplifying assumption I make concerns the wear and tear of fixed capital caused by its use in production. I assume that this "extra wear and tear" is negligible and thus the total wear and tear is due to obsolescence. This assumption, contrary to the Keynesian conception of user-cost, does not imply an underestimation of its importance but is simply made to avoid complications inherent in this subject. I think, however, that this simplification will not affect the results of our analysis much.

With this assumption, the only prime costs are those of labour and raw materials. If we thus denote by the income of capitalists from an enterprise the difference between the value of its output and the value of prime costs, we find that this income is equal to the value of production minus the cost of labour and raw materials. We shall call the national income the sum of capitalists' and workers' incomes. It is easy to see that the national income is equal to the sum of the value of the output of all enterprises minus the value of the output of raw materials. But, hence, it follows that the national income is equal to the value of consumption, purchases of fixed capital equipment, and increase of stocks. The value of the purchases of fixed capital and the increase of stocks we shall call investment. It is clear that this is gross investment and also that the income of capitalists means here their gross income, i.e. that from either the supplementary cost is not subtracted.

For both Keynes' theory and this paper the notion of a given capital

equipment is essential. The objection is often raised that it is wrong to assume a given capital equipment within a period, because the investment changes the equipment during this period. The answer is very simple: this period can be made so short that the change in the equipment is small enough not to affect the formation of output and income. For output and income are quantities measured per unit of time and thus are not dependent on the length of the period taken into consideration, whilst the change of equipment is, *caeteris paribus*, proportionate to this length.

### SHORT-PERIOD EQUILIBRIUM

1. Output with a given capital equipment depends on the quantity of labour employed and on its distribution among the various sections of this equipment. In every enterprise the employment is pushed to the point at which marginal revenue is equal to the marginal prime cost.

We shall represent the point of intersection of the marginal revenue and the marginal prime cost curves as follows. We subtract from both price and prime costs the cost of raw materials, and thus we obtain so-called value added and labour costs respectively. We can now say that the output of an enterprise is given by the intersection of the curves of marginal value added and of marginal labour-cost (see Fig. 1). Marginal value added and marginal labour-cost are both expressed here in wage units.<sup>1</sup> We shall call short-time equilibrium a state in which the marginal labour-cost curves and marginal value-added curves do not move. With a given capital equipment the curves of marginal labour-cost are fixed. The establishment of short-time equilibrium with a given equipment will thus consist in the shift of marginal value-added curves.

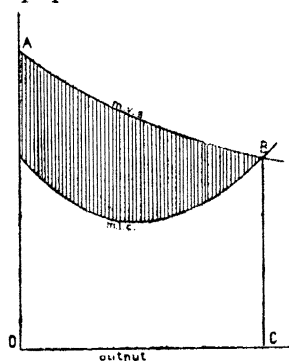


FIG. 1.

The area  $OABC$  is the value added of the enterprise expressed in wage-units, the hatched area is the income of the capitalists obtained from this enterprise, while the unhatched area is the income of the workers. Thus the sum of  $OABC$ —areas of all enterprises—is the national income expressed in wage-units, while the sum of the hatched areas is the total income of the capitalists, and that of the unhatched areas the total income of the workers. The national income is also equal to the value of total consumption and total investment and, as we have assumed that the workers do not save, the sum of the unhatched areas covers the value of the consumption of the workers, while the sum of the

<sup>1</sup> Keynes defines the wage-unit as follows: "... in so far as different grades and kinds of labour and salaried assistance enjoy a more or less fixed relative remuneration, the quantity of employment can be sufficiently defined for our purpose by taking an hour's employment of ordinary labour as our unit and weighting an hour's employment of special labour in proportion to its remuneration; i.e. an hour of special labour remunerated at double ordinary rates will count as two units. We shall call the unit in which the quantity of employment is measured the labour-unit; and the money-wage of a labour-unit we shall call the wage-unit." *The General Theory of Employment, Interest and Money*, p. 41.

hatched areas is the value of the consumption of the capitalists and of the investment.

We can now make clear the key position of the spending of the capitalists in the formation of short-time equilibrium. In such an equilibrium the marginal value-added curves remain by definition in a certain determined position. As we have just shown, the sum of the hatched areas is equal to the spending of the capitalists on consumption and investment, and the sum of the unhatched areas covers the consumption of the workers. There can be no spontaneous change in the spending of the workers, because they spend, by assumption, as much as they earn, but such changes of spending are quite possible for capitalists. Let us assume that the capitalists spend a given amount more than before per unit of time. Then there will be a shift in the marginal value-added curves until the sum of the hatched areas is equal to the increased spending by the capitalists for consumption and investment. As the sum of the hatched areas is also equal to the total income of the capitalists, the increase of their spending "forces" their income to rise by the same amount.

It is clear that in the new short-period equilibrium the employment, the income of the workers, and therefore the value of their consumption (measured in wage units), is greater than before. Hence, it follows that the demand for all kinds of investment and consumption-goods, for both capitalists and workers, has risen, and thus a shift of the marginal value-added curves must have taken place in all industries.

We see now that the spending of the capitalists determines a position of marginal value-added curves such that the sum of the hatched areas, i.e. the incomes of the capitalists, is equal to the amount they spend. In this way the level of the spending of the capitalists (expressed in wage-units) is the chief determinant of the short-period equilibrium and particularly of employment and income.

2. We have shown that the spending of the capitalists "forces" a capitalists' income which is equal to this spending. As the spending of the capitalists consists of their consumption and investment, and the income of the capitalists of their consumption and saving, it can also be said that the investment "forces" saving to an amount which is equal to the amount of this investment. It is clear that in general the same capitalists do not invest and save: the investments of some create a saved income of an equal amount for others.

We assume now a definite capitalists' propensity to consume, i.e. to every level of total capitalists' income expressed in wage-units there corresponds a definite distribution of this income between consumption and saving. It is clear that in this way to every level of saving there corresponds a definite level of capitalists' consumption. Hence, it can easily be concluded that the amount of investment expressed in wage-units determines the total sum of the spending of capitalists. For the amount of investment  $I$  "forces" an equal amount of saving, and if capitalists' consumption is, say, lower than the level  $C$  corresponding to the amount  $I$  of saving, then the capitalists will consume more; in this way they will "push" their income to the level  $C + I$

at which the proportion between the consumption  $C$  and saving  $I$  is in accordance with their propensity to consume.

3. We see now that the total investment  $I$  per unit of time expressed in wage-units determines, *grosso modo*, the short-period equilibrium. For with a given propensity to consume there corresponds to  $I$  a definite capitalists' consumption  $C$ , and thus we have the total spending of the capitalists  $C+I$  and its distribution between consumption and investment. To determine the short-period equilibrium in full detail we need, in addition to this, some knowledge of the kind of investments and the "tastes" of both capitalists and workers. If we assume these "tastes" as known, the only indeterminate element in the short-period equilibrium corresponding to the given amount  $I$  of spending on investments (in wage-units) per unit of time is the distribution of this spending amongst various kinds of investments. But we can admit, I think, that the changes in the structure of investment expenses have no great importance for the general employment and national income  $Y$  expressed in wage-units, and we can write, therefore, without making a considerable mistake :

$$Y = f(I).$$

$f$  is here an increasing function and its shape is defined by the given capital equipment, capitalists' propensity to consume, and the "tastes" of capitalists and workers. The derivative of this function :

$$\frac{dY}{dI} = f'(I)$$

is the Keynesian multiplier. If investment changes from the *given* level  $I$  to the *given* level  $I + \Delta I$ —where  $\Delta I$  is a small increment—then income will change from the level  $Y$  to the level  $Y + \Delta I \cdot f'(I)$ . This is the only question the multiplier answers and no other service can be required from it.

## THE DYNAMIC PROCESS AS A CHAIN OF SHORT-PERIOD EQUILIBRIA

1. With given capital equipment, capitalists' propensity to consume, and the "tastes" of both capitalists and workers, the amount of investment  $I$  expressed in wage-units determines, as we have seen above, almost entirely the short-period equilibrium (the only indeterminate factor being the structure of investment) and particularly the amount of total employment and income. Thus it can now be asked : "What determines investment?" Here a treatment of the subject called by Keynes "inducement to invest" might be expected, but we postpone the examination of this problem to the next section, and now we propose to consider the matter from quite a different point of view. We wish now to state that the present investment, i.e. the value of present investment output, is a result not of *present* but *former* investment decisions,

for, as we shall see immediately, a certain relatively long time is needed to complete the investment projects. This fact is of fundamental importance for the dynamics of an economic system. For the investments at a given moment fail to be a variable dependent on other factors acting at this moment and become a *datum* inherited from the past like the capital equipment. (We assume that the investment decision is irrevocable in the course of the construction of the particular object.) It is clear that the present phenomena are also a basis for investment decisions, which, however, will be relevant for the formation of investment output only in the future, and so on.

2. Let us now examine more closely the dependence of present investment output on former investment decisions. If it is known that two years, say, are needed to build a factory, then during two years from the moment of the investment decision  $1/24$  of this factory will be produced monthly. Now it is easily seen that the output of investment per unit of time is determined by the set of investment orders not yet completed and the corresponding time-spaces necessary to finish them. If, for instance, at the beginning of a month the building of a factory worth £1,000,000 is ordered for delivery twenty months hence; and besides this there remains to be completed half a factory the total value of which is £1,200,000 and time of building twelve months; then the value of orders to be finished is £1,000,000 and £600,000 respectively, whilst the time needed is twenty months and six months; thus the monthly investment output is  $\frac{£1,000,000}{20} + \frac{£600,000}{6} = £150,000$ . No difficulty arises in generalising this formula. If we denote the parts of the investment orders not yet completed (reckoned at prices current at the given moment expressed in wage-units) by  $o_1, o_2, \dots$  and the corresponding time needed by  $\tau_1, \tau_2, \dots$  the present level of investment is:

$$I = \sum \frac{o_k}{\tau_k}$$

Let us now denote the sum  $\sum o_k$  of uncompleted parts of investment orders at a given moment by  $O$ . We define as the average time  $\tau$  a time such as is needed to produce investment goods of the value  $O$  at a rate of investment

$I = \sum \frac{o_k}{\tau_k}$ . Thus we have:

$$\tau = \frac{O}{I} = \frac{\sum o_k}{\sum \frac{o_k}{\tau_k}}$$

It should be pointed out that  $\tau$  is not the average time required for the completion of investment decisions (gestation period), but the average time required for finishing the orders, which are in diverse stages of construction; in this way  $\tau$  is, roughly speaking, half the average gestation period, because at every moment there exist orders whose completion has just begun, is near to the end, or has reached a position intermediate between these two extreme

cases. In reality  $\tau$  is likely to be equal to a few months; it is certainly not a constant value, but it varies slowly within a narrow range (see the mathematical note at the end of this paper). We shall assume for the sake of simplicity that  $\tau$  is constant; but it is easy to see that the argument can be reconstructed without any difficulty for the case of slowly varying  $\tau$  (see footnote 1 below). From the above equation it follows  $I = \frac{0}{\tau}$ , thus if  $\tau$  is assumed to be constant the rate of investment is proportionate to the value of the "stock" of uncompleted orders.

3. Let us now imagine that the time is divided into periods of the length  $\tau$ , supposing that within every one of these periods the investment  $I$  does not change, i.e. that instead of a continuous time curve we consider a "stair line" inscribed in this curve (see Fig. 2). In a similar way we imagine also that the change of capital equipment in a  $\tau$ -period does not affect the short-time equilibrium in this period but in the next. Thus as investment and capital equipment nearly define the short-period equilibrium-output, income and prices also will prevail at a definite level during a  $\tau$ -period and will change at the end of it.

At the beginning of period 1 we have a certain stock of uncompleted projects. The investment  $I$  in that period is equal to its current value divided by  $\tau$ . Thus the value of investment goods  $I_1\tau$  produced in the period is just equal to the value of uncompleted parts of investment orders at the beginning of the period. Consequently the "stock" of uncompleted investment projects at the end of the  $\tau$ -period is equal to the amount of investment decisions undertaken during this period.

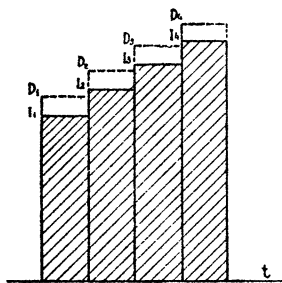


FIG. 2.

If we thus denote the investment decisions per unit of time, i.e. the rate of investment decisions in period 1 by  $D_1$  (reckoned at prices current during the period 1 in wage-units), the sum of uncompleted investment orders at the end of the period is equal to  $D_1\tau$ . But the investments per unit of time in the 2-period reckoned at prices of period 1 are equal to this carry-over of orders from the first period divided by  $\tau$  or  $\frac{D_1\tau}{\tau} = D_1$ , i.e. they are equal to the rate of

investment decisions in the first period.<sup>1</sup> Thus, if (as

on the chart)  $D_1 > I_1$ , the investment in the period

2 reckoned at the prices of period 1 are larger than in the period 1. This increased demand for investment goods will increase their prices (by an amount dependent on the state of equipment in investment-good industries) and consequently the value of investments in period 2.  $I_2$  is thus, in turn,

<sup>1</sup> The assumption about the constancy of  $\tau$  was needed in order to demonstrate this proposition. But it is clear that it would be enough to assume that the difference between the length of two  $\tau$ -periods following each other is negligible. Thus the assumption of the slow variation of  $\tau$  is necessary but not its constancy. The latter is made only to simplify the exposition.

greater than  $D_1$ . We have, consequently, the inequality :

$$I_1 < D_1 < I_2$$

The difference  $D_1 - I_1$  results from the "real" increase of investment between the periods 1 and 2, whilst the difference  $I_2 - D_1$  is due to the rise in prices of investment-goods.<sup>1</sup> If  $D_1$  were less than  $I_1$  the inequality would have changed its direction. Whilst if  $D_1 = I_1$  the investment  $I_2$  would also be equal to  $I_1$ .

4. It follows from the above that the amount of investment  $I$  (measured in wage-units) in a given  $\tau$ -period is determined by the rate of investment decisions in the preceding period. Thus we can now imagine the dynamic process as a chain of short-period equilibria each of them prevailing during time  $\tau$ . Suppose we have in the initial  $\tau$ -period a given amount of investment  $I_1$  expressed in wage-units, which on the basis of the capital equipment determines a short-period equilibrium. This state, which can be represented by the set of marginal value-added curves and marginal labour-cost curves of all enterprises, in conjunction with some other factors (principally the rate of interest), defines the rate of investment decisions  $D_1$  in this period. But these decisions in turn determine the investment  $I_2$  in the period 2 and in that way also the new short-period equilibrium on the basis of the capital equipment, which has, of course, also changed in general as a result of wear and tear and of investments in the preceding period. Thus there is a new level of investment decisions and a further change in capital equipment caused by its wear and tear and by investments in the second period. As a result we have again a new short-period equilibrium in the next period, and so on.

To be able to say more about the mechanism of the dynamic process we must now examine the motives of investment decisions in order to show how the links of our chain are connected.

## THE INDUCEMENT TO INVEST

1. In the Keynesian theory of inducement to invest the fundamental notion is that of the marginal efficiency of an asset. Keynes defines it as the rate at which the prospective current returns (differences between revenues and effective expenditures) of an asset during its future "life" have to be discounted in order to obtain the present supply price of this asset. Keynes assumes that the greater the investment in a certain type of capital per unit of time the less will be the marginal efficiency of the corresponding assets because of the rise of the supply prices of these assets. "Now it is obvious that the actual rate of current investment will be pushed to the point where there is no longer any class of capital of which the marginal efficiency exceeds the current rate of interest."<sup>2</sup> In other words, if at a given moment there is

<sup>1</sup> As the result of changes in the prices of investment goods a difference between the value of produced investment and the value of the corresponding orders will in general arise. Thus, if the prices have, say, risen, the entrepreneurs, who had given orders and obtain the investment objects at the "old" prices, make a relative gain, whilst the producers, whose marginal prime cost have undergone in the meantime a rise, suffer a relative loss.

<sup>2</sup> *General Theory*, p. 136.



a gap between the marginal efficiency of the various assets and the rate of interest, the investment per unit of time will rise until the increase of the prices of investment goods caused by this will reduce the marginal efficiency of all assets to the level of the rate of interest.

There are two things lacking in this conception. First it tells us nothing about the rate of investment decisions taken by entrepreneurs faced with *given* market prices of investment goods. It indicates only that unless the marginal efficiency of all assets calculated on the basis of this level of prices of investment goods is equal to the rate of interest, a change of investment will take place which will transform the given situation into a new one, in which the marginal efficiency of various assets *is* equal to the rate of interest.

But a new trouble now arises. Let us assume that the rate of investment has really, say, risen so much that the new level of investment prices and the *initial* state of expectations give a marginal efficiency equal to the rate of interest. The increase of investment, however, will cause not only the prices of investment goods to rise, but also a rise of prices (or, more precisely, the upward shift of marginal revenue curves) and employment in all branches of trade. Thus, because "the facts of the existing situation enter, in a sense disproportionately, into the formation of our long-term expectations,"<sup>1</sup> the state of expectations will improve and the marginal efficiency of assets appears again higher than the rate of interest. Consequently "equilibrium" is not reached and the investment continues to rise.

We see now that the Keynesian conception, which tells only how great investment will be if the given "disequilibrium" changes into an "equilibrium," encounters a difficulty in this respect also, for it appears that the rise of investment does not lead to "equilibrium" at all (in any case, not to immediate "equilibrium"). I shall further try to give an outline of a different conception of inducement to invest which endeavours to find factors determining the amount of investment decisions corresponding to *every* definite state of long-term expectations, prices of investment goods, and rate of interest.

2. We start from the problem of uncertainty, which is also involved in Keynes' arguments. It can be gathered from his exposition that a certain amount has to be subtracted from the marginal efficiency of assets (calculated on the basis of the current prospective returns) to cover risk before comparing it with the rate of interest. We can express the same point in this manner: the gap between the marginal efficiency of assets calculated on the basis of the prospective current returns of these assets, which we shall call the prospective rate of profit, and the rate of interest, is equal to the risk incurred. But here we wish to draw attention to a point not considered by Keynes.

*The rate of risk of every investment is greater the larger is this investment.* If the entrepreneur builds up a factory he incurs a certain risk of unprofitable business, and these losses, if any, will be more significant for him the greater proportion the investment considered bears to his wealth. But besides this, in "sacrificing" his reserves (consisting of deposits or securities) or taking credits, he exhausts his "sources of capital," and if he should need this

<sup>1</sup> *General Theory*, p. 148.

"capital" in the future he may be obliged to borrow at a high rate of interest because he has overdrawn the amount of credit considered by his creditors as "normal." Thus both these aspects of risk incurred by investment show that the rate of risk must grow with the amount invested.

Now, I think we have the key to the problem of amount of investment decisions in a given economic situation in a certain period of time, for instance, in our  $\tau$ -period. This amount is just so much as will equate the marginal risk to the gap between the prospective rate of profit and the rate of interest, both being given by the economic situation of the period in question. The greater the "gap" the greater is the sum of investment decisions in the period, and this for two reasons. First the number of people undertaking investment increases, including the more timid entrepreneurs; and, secondly, each of them invests more.

3. In all this conception, however, an obscure point still remains. The entrepreneurs in the  $\tau$ -period considered have taken so many investment decisions that any additional investment decision does not seem to them sufficiently attractive because of the growing risk. Will there, then, be no investment decision at all in the next  $\tau$ -period if the gap between the prospective rate of profit and rate of interest remains at the same level as before? Certainly this is not the case. For the value of the investment in the second period—as we know from the preceding chapter—corresponds to the investment decisions in the first  $\tau$ -period; further, the saving in the second period is equal to the investment in the second period; thus the capitalists as a body save in the second  $\tau$ -period just the amount which they decided to invest in the first  $\tau$ -period. To the money-flow of investments there corresponds an equal money-flow of savings, and if investment decisions of an equal amount should not be taken, an improvement in the security of wealth and liquidity for the entrepreneurs would result (who accumulate reserves or repay debts) at the end of the period; hence, the marginal risk would be less than the gap between prospective rate of profit and the rate of interest. In this way if the gap remains as supposed on the same level, a steady reinvestment of the same amount will take place. The flow of investment decisions continuously imposes the burden of risk on some capitalists, but the equal flow of savings relieves other capitalists from this burden.

If the gap between the prospective rate of profit and the rate of interest increases, the investment decisions in a  $\tau$ -period will be pushed to the point at which the marginal risk is equal to the increased gap. If this gap does not change further, reinvestment of the new higher amount will take place in the following periods.

Thus we can now say that *the rate of investment decisions is an increasing function of the gap between the prospective rate of profit and the rate of interest.*<sup>1</sup>

<sup>1</sup> This can also be deduced as follows. It can be concluded from the above that the burden of risk is created only by the existence of unrealised investment decisions. Thus this burden is, *ceteris paribus*, higher the larger the "stock" of uncompleted orders at the end of a given  $\tau$ -period, which (see p. 82) is equal to  $Dr$ . Or the marginal risk increases with the rate of investment decisions  $D$  and, consequently, so must the gap between the prospective rate of profit and the rate of interest needed to cover the risk.

## TWO DETERMINANTS OF THE INVESTMENT DECISIONS

1. We have shown that the rate of investment decisions is an increasing function of the difference between the prospective rate of profit<sup>1</sup> and the rate of interest. To find out the determinants of investment decisions we must analyse the factors on which this difference is dependent. We shall divide the analysis into two parts: in the first we assume a given capital equipment, in the second we examine the effects of changes in this capital equipment.

The prospective rate of profit is defined by the long-term expectations of returns and the supply prices of investment goods. It was mentioned above that the point of departure for estimating future returns is first of all the present state of affairs. Thus it is the short-period equilibrium which chiefly determines the prospective rate of profit at the given moment. For in this short-period equilibrium we have given the system of marginal value-added curves, which describes "the present state of affairs," while these curves and the marginal labour-cost curves in the investment-good industries give us the level of investment-good prices.

But with a given capital equipment the short-period equilibrium is determined by the rate of investment  $I$ , and so, consequently, is the prospective rate of profit. The change in the rate of investment  $I$  will affect the prospective rate of profit from two sides in opposite directions: the increase, say, of the investment will raise the marginal value-added curves and consequently improve expectations, but at the same time it will also increase the prices of investment goods. Thus we can say that the prospective rate of profit with a given capital equipment is a function of investment  $I$ , but we do not know, *a priori*, whether this function is increasing or decreasing.

2. We are now going to show that with certain assumptions the rate of interest can also be represented as a function of investment  $I$ . We know that with a given capital equipment both employment and the national income  $Y$  expressed in wage-units are increasing functions of  $I$ . Here we shall also make a justifiable assumption that with the rise of employment the wage-unit  $w$  increases in a definite way (due to a relative shortage of certain kinds of labour, improvement in the position of trade unions and so on). Thus income expressed in terms of money  $Yw$  will increase in a definite way if the investment  $I$  rises. For the rise of  $I$  causes a rise of  $Y$ , while the increased employment pushes nominal wages to a higher level.

The greater the money income  $Yw$  the greater is the demand for cash for transactions, which, with a constant amount of money in circulation, must cause the rate of interest to increase. In general, the amount of money in circulation will not remain constant because the banking system creates new money; but also, in that case, we can assume that this creation will be accompanied by a rise in the rate of interest because of the falling liquidity of banks.

<sup>1</sup> It is clear that in general the prospective rates of profit in various industries are not equal. But we can define the general prospective rate of profit as such a rate which, if it were to prevail in all industries, would affect the rate of investment decisions in the same way as the given set of prospective rates of profits.

We see thus that the rise of investment  $I$  increases the demand for cash and has in that way the tendency to raise the rate of interest. It is, however, not the only way in which the rate of interest is affected by change in investment  $I$ . The investment  $I$  as we know determines (with a given capital equipment) the short-period equilibrium and thus the "general state of affairs." But the better this state of affairs the greater is the "lender's confidence" <sup>1</sup> and, therefore, through this channel the rise of investment has a tendency to lower the rate of interest.

Probably these two opposite stimuli will cause the rate of interest to fall initially with an increase of investment  $I$ , but after passing a minimum the rate will begin to rise when investment further increases. For at a low level of investment  $I$ , and thus of income  $Y$ , the elasticity of supply of money is high, while an improvement in business much affects the "lender's confidence," and thus the rate of interest is likely to fall with the rise of investment. But at a high level of investment and income, as the supply of money has become more inelastic and the "lender's confidence" is less sensible to a further rise in business activity, the increase of investment will rather cause the rate of interest to rise.

3. We have stated that both the prospective rate of profit and the rate of interest can be represented with certain assumptions as functions of investment  $I$ . Thus the rate of investment decisions which is an increasing function of the difference between the prospective rate of profit and the rate of interest is also the function of investment  $I$ .

$$D = \phi(I)$$

Hence, it follows that in a given  $\tau$ -period it is the level of investment which determines the rate of investment decisions and thus the investment in the next  $\tau$ -period.

We cannot say *a priori* whether the function  $\phi$  is increasing or decreasing. For the rise of  $I$  improves the expectations of returns, but at the same time raises the prices of investment goods and may also raise (if  $I$  is sufficiently great) the rate of interest. But it is very probable that below a certain level of  $I$  this function *is* increasing. For if the level of investment is not relatively high the marginal prime cost curve in the investment-good production is only slightly increasing with output and, consequently, so are the prices of investment goods. The rate of interest which initially falls with the increasing investment also after having passed the minimum within a certain interval rises only slightly. Thus, before  $I$  reaches a certain rather high level it can be assumed that a rise in it affects investment decisions more by improvement of expectations than by raising prices of investment goods and the rate of interest.

We can now discover some further features of the function  $\tau$  which is represented here in Fig. 3. We shall try to show that the curve  $MAN$  representing this function must cut the straight line  $OL$ , drawn at  $45^\circ$  through the zero point  $O$ , and that the left part  $MA$  lies above, whilst the right

<sup>1</sup> See on the "lender's confidence," *General Theory*, pp. 144, 309.

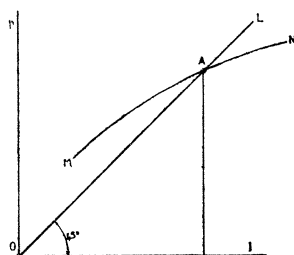


FIG. 3.

part  $AN$  lies below  $OL$ . In other words, there exists a value of investment  $I_A$  to which corresponds a value of investment decisions  $D_A$  equal to  $I_A$ , while for investment lower than  $I_A$  we have  $D > I$ , and for investments higher than  $I_A$  the opposite, i.e.  $D < I$ . There are, *a priori*, three possible positions of the curve  $\phi$  besides that shown in Fig. 3 (see Fig. 4). We shall show that they are unrealistic. It is easy to show that if the curve lies entirely above  $OL$ , or, which is the same, if  $D$  is always greater than  $I$ , we shall have an unlimited cumulative upward process. For if in a certain  $\tau$ -period there corresponds

to investment  $I$  a higher amount of investment decisions  $D$ , then in the next  $\tau$ -period the investment will be higher; but because the curve  $\phi$  lies above

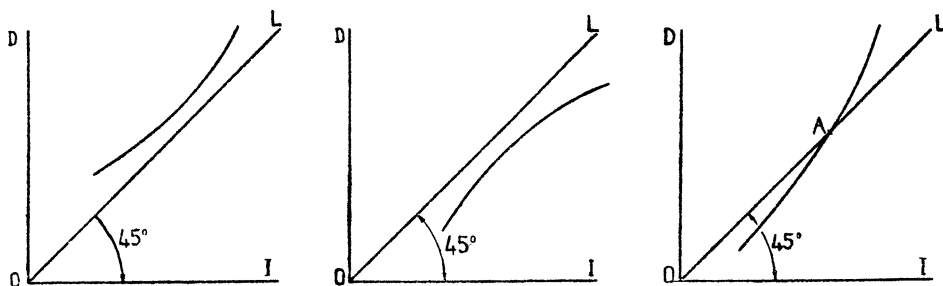


FIG. 4.

$OL$  the investment decisions in the second  $\tau$ -period are again higher than the investment, and so on. In that way the investment would increase automatically without limit.

This is, however, impossible, for the limited amount of available labour does not permit investment and income to pass a certain level. What is the mechanism by which the cumulative process is stopped? In the neighbourhood of full employment the rise of nominal wages corresponding to every small increase of investment (measured in wage-units) will be very sharp. It will cause a rapid rise of nominal income, of demand for money, and thus of the rate of interest. In that way the latter will soon reach the level at which investment decisions are equal to investment and thus there will be no tendency for a further rise of investment. But it all amounts to nothing more than the demonstration of the feature in question of the function  $\phi$ . Because of the rapid rise of the rate of interest with the increased investment in the neighbourhood of full employment, the shape of this function must be such that the curve  $MAN$  cuts the straight line  $OL$  in a point, which cannot lie above the investment level corresponding to full employment. But it is clear that it may lie lower. For the investment in successive  $\tau$ -periods may form a convergent series even without the restraining influence of the rate of interest.

4. We shall now demonstrate that the curve  $MAN$  can not lie entirely below the straight line  $OL$ . In that case we should have an unlimited cumulative downward process. For if in a certain  $\tau$ -period there corresponds to investment  $I$  a lower amount of investment decisions, it will cause a lower level of investment in the next period ; but in that period  $D$  is again lower than  $I$  and thus the downward process goes on. But, as in the case of the upward process, an unlimited movement is again impossible, though the factor which determines the limit is of quite a different nature.

The investment  $I$  by our definition is the value (expressed in wage-units) of the purchases of fixed capital and the increase of stocks per unit of time. Thus it can be negative if the decrease of stocks is greater than the purchases of fixed capital, but, as we shall show at once, this negative value cannot fall below a certain level. We know that the capitalists' income is equal to their spending  $C+I$  for consumption goods and investment. This income (from which supplementary costs are not subtracted) cannot be lower than zero, for otherwise the entrepreneurs would not produce at all. Thus we find that  $C+I \geq 0$  and, consequently,  $I \geq -C$ . Or the curve  $MAN$  must cut the straight line  $OL$  in a point at which  $I$  is not lower than  $-C$ , where  $C$  is the capitalists' consumption in the case when their income is zero.

Now it is easy to see that the third position of the curve also is unrealistic ; for if the investment is initially lower than the abscissa of  $A$  we have an unlimited downward cumulative process, and if it is initially higher than the abscissa of  $A$  an upward cumulative process goes on indefinitely. To summarise : We have stated three features of the function  $\phi$  represented by the curve  $MAN$  :

(1) The curve  $MAN$  is initially ascending.

(2) This curve cuts at point  $A$  the straight line  $OL$  drawn through the zero point at  $45^\circ$ . The part  $MA$  of  $MAN$  lies above and the part  $AN$  below  $OL$ .

(3) The investment  $I$  at this point of intersection  $A$  with  $OL$  is not higher than the investment level corresponding to full employment and not lower than  $-C$ , where  $C$  is capitalists' consumption, when their income is zero.

5. We have up till now examined the dependence of the rate of investment decisions  $D$  on investment  $I$  assuming a given capital equipment. Now we are going in turn to analyse the influence of changes in this equipment on the investment decisions if the investment is given. In that way we shall be able to describe  $D$  as a function of both investment  $I$  and capital equipment.

We begin with the statement that if investment  $I$  remains constant (capitalists' propensity to consume assumed as given) so also does the total capitalists' spending  $C+I$  and, consequently, the total capitalists' income, which is equal to their spending. Thus, if the capacity of equipment, say, increases, it is easy to see that the "state of affairs" becomes worse. For if the same income is earned by capitalists on a greater number of factories the income on every factory is less. The "new" factories compete with the "old" ones, the downward shift of marginal value-added curves reduces the capitalists' income (hatched area on Fig. 1) in the "old" factories, and

in that way a part of the total income of the capitalists  $C + I$ —being by assumption constant—is transferred to the “new” factories.

Thus it is clear that the increase of capital equipment with constant investment  $I$ , and thus with the constant spending and income of the capitalists, must have a depressing effect on expectations. It is not certain, however, whether the prospective rate of profit will fall; for if the equipment is expanded also in investment-good industries the prices of these goods will decline and this may counterbalance the less favourable state of expectations.

We abstract this case, however, from further exposition for the sake of simplicity<sup>1</sup> and thus assume that with the constant spending of the capitalists the expansion of equipment causes the prospective rate of profit to fall.

The depressing effect of the increase of equipment on the prospective rate of profit stated here is also one of the fundamental propositions of the Keynesian theory. But it is considered there rather as a general principle which does not require to be proved. From our above argument it is clear that this law is valid only on the assumption of the constant spending of the capitalists (and in that case also with some additional assumption); if this spending increases in the same proportion as equipment the prospective rate of profit has no tendency to fall.

Our proper aim was to state the influence of the change in the capacity of equipment on the investment decisions when the spending of the capitalists remains constant. The investment decisions are, as we know, an increasing function of the gap between the prospective rate of profit and the rate of interest. We have stated that (on certain assumptions) the prospective rate of profit falls when equipment is expanding. We have yet to examine what will happen to the rate of interest.

If the equipment expands with the constant spending of the capitalists, the marginal value-added curves shift down, and the degree of employment in each factory diminishes. But this is accompanied by the fall of the relative share of the capitalists in value added in each factory<sup>2</sup> and, consequently, of the relative share of the capitalists in the national income. Since, however, their income, which equals their spending, is by assumption constant, this means that the national income must increase. Thus the expansion of equipment with the constant spending of the capitalists causes a rise of demand for cash and, consequently, an increase of the rate of interest.

<sup>1</sup> It can be shown that this simplification does not affect the validity of the explanation of the business cycle given in the next section. The case abstracted can occur only on the top of the boom when the supply of investment goods may become inelastic, because only on that condition will the increase of equipment producing these goods cause their prices to fall significantly. We should have then a situation in which investment does not rise (because it is the *top* of the boom), equipment expands, and the prospective rate of profit does not fall. This situation, however, could not last long. For the fall of the prices of investment goods would continuously increase the profitability of consumption-good industries at the expense of investment-good industries. Thus there would be a shift of investment activity from the latter to the former, the increase of consumption-goods equipment would be accelerated and that of investment-goods equipment retarded; and this would cause the expected returns to fall more strongly than the prices of investment goods. The fall of the prospective rate of profit—which in our representation of the business cycle process in the next section accounts for the breaking down of the boom—would only be delayed; the economic system would stay longer on the top of the boom, to be, however, eventually overcome by the slump.

<sup>2</sup> This is not strictly a rule, but the opposite case can be considered exceptional.

From this and the depressing influence on the prospective rate of profit it may be concluded that the increase of the capacity of equipment with the constant spending of the capitalists causes a fall of the gap between the prospective rate of profit and the rate of interest, and thus a fall of investment decisions. But if the investment is constant the spending of the capitalists is constant, too. Or we get : the greater the equipment with a constant investment  $I$ , the less the rate of investment decisions  $D$ .

The curve representing the function  $D = \phi(I)$  is drawn on the assumption of a constant equipment. If the equipment changes, this curve will be shifted. And it follows from the above that it will be shifted down when the equipment increases. The greater the capacity of the equipment, the lower will be the position of the curve  $\phi$ . In that way the family of curves  $\phi$  represents the rate of investment decisions  $D$  as a function of two determinants—the rate of investment  $I$  and the equipment.

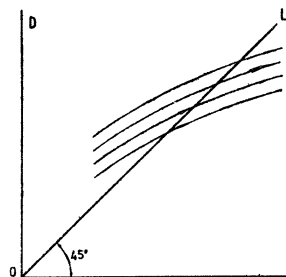


FIG. 5.

### THE BUSINESS CYCLE

1. Let us now, again, consider the dynamic process represented as a chain of short-period equilibria, each lasting a  $\tau$ -period. To simplify the exposition we will examine this process in two stages : in the first we abstract the changes of capital equipment ; in the second stage we take into account also the influence of the changes which result from investment and wear and tear.<sup>1</sup>

Suppose the level of investment (expressed in wage-units) in the first  $\tau$ -period to be  $I_1$  (see Fig. 6).

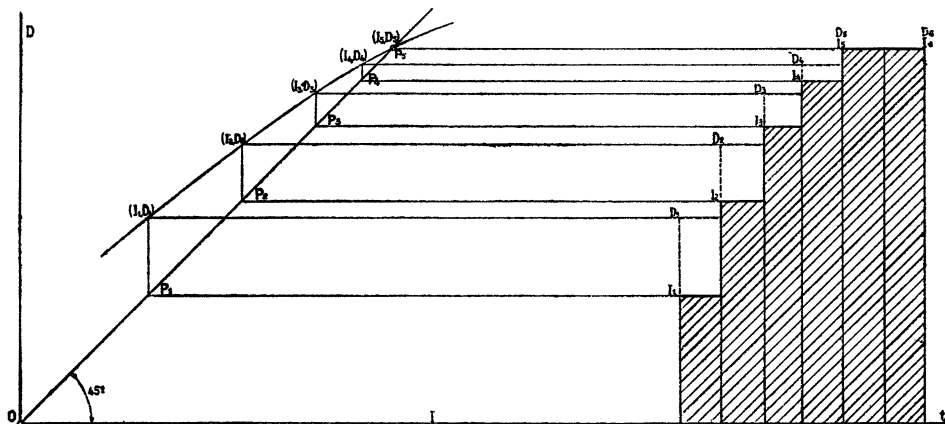


FIG. 6.

<sup>1</sup> In the first stage we can imagine, for instance, that both investment and wear and tear are very small in relation to equipment ; thus the equipment changes only a little in the course of the process considered.



The curve  $\phi$  on the left represents the dependence of the rate of investment decisions  $D$  on the investment  $I$  with a given equipment. Drawing a horizontal on the level  $I_1$  we obtain first the point of intersection  $P_1$  with  $OL$ , whose abscissa (being equal to the ordinate) is equal to  $I_1$ . Drawing the vertical through  $P_1$  we obtain on the curve  $\phi$  the point  $(I_1, D_1)$ , whose ordinate is  $D_1$ —the rate of investment decisions corresponding to  $I_1$ —and thus that which will take place in the first  $\tau$ -period.

The rate of investment decisions in the first  $\tau$ -period is, as we see, higher than investment (we have so chosen our initial position). We know from the second section that the investment in the next  $\tau$ -period reckoned at the prices of the first  $\tau$ -period is equal to  $D_1$ . Thus, because  $D_1 > I_1$  the "real" value of investment in the second  $\tau$ -period is greater than in the first; this causes the prices of investment goods to increase, and we have:

$$I_1 < D_1 < I_2$$

where  $D_1 - I_1$  is the "real" increase of investment from period 1 to period 2, and  $I_2 - D_1$  is due to the corresponding rise of prices of investment goods.

Now, with the help of the curve  $\phi$ , we can obtain again the level of investment decisions  $D_2$ , which is again greater than  $I_2$  and which causes the increase of investment to the level  $I_3$  in the third period, and so on.

We reach, finally, in that way in the fifth period the level of investment  $I_5$ , to which there corresponds on the curve  $\phi$  the point of intersection of this curve with the straight line  $OL$ , i.e. we reach a position in which  $D_5 = I_5$ . Thus from this very moment the investment ceases to grow and in the sixth  $\tau$ -period the investment remains on the same level and so also does the rate of investment decisions, which is equal to investment. All the process can be represented by the following scheme:

$$I_1 < D_1 < I_2 < D_2 < I_3 < D_3 < I_4 < D_4 < I_5 = D_5 = I_6 = D_6$$

We see here that the excess of investment decisions over investment in the first period causes a self-stimulating rise of investment, which in its essential nature is identical with the so-called Wicksellian cumulative process. This rise, however, leads to a position in which the investment ceases to grow, remaining afterwards at a constant level. (This maintenance of investment after period 5 takes place only on the assumption of constant equipment; we shall see in the next paragraph that it is precisely the increase of equipment, which disturbs this "equilibrium.") It is easy to see that the "equilibrium" reached in period 5 is stable; if the investment is lowered beneath the level  $I_5$  we shall have a rise represented above bringing it back to this level. But, also, if it rose above this value a fall of investment would take place and push it again back to the "equilibrium" level; for it is clear, in general, that if we start from a position in which  $D < I$  we shall obtain a downward cumulative process in exactly the same way as we constructed the upward one above.

With a given curve  $\phi$  the time of adjustment leading to the state of "equilibrium" is proportionate to the length of the  $\tau$ -period. In general the time of change of investment from one given level to another with a given curve  $\phi$  is proportionate to  $\tau$ .

It is worth noting that these cumulative processes have nothing (at least directly) to do with the Keynesian multiplier. This last answers only the question of how much the national income will increase from a certain  $\tau$ -period to the next  $\tau$ -period as a result of the increase of investment; while the mechanism of the cumulative process determines this growth of investment as such. We can represent this by the following scheme:

$$\begin{array}{ccccccc} I_1 & < & I_2 & < & I_3 & < & I_4 & \dots & \dots \\ | & & | & & | & & | & & \\ Y_1 & < & Y_2 & < & Y_3 & < & Y_4 & \dots & \dots \end{array}$$

2. We come to the second stage of our analysis of the dynamic process: we have now to consider the influence exerted on the course of this process by changes which the capacity of equipment undergoes.

Thus, if the upward cumulative process described above starts from a position in which the investment is lower than the "level of maintenance"  $W$ , the change of equipment stimulates it. But the situation alters when the investment begins to exceed the level of maintenance of capacity. The equipment capacity is expanding and this retards the cumulative process. Or, in other words, the curve  $\phi$  along which the point  $(I, D)$  moves shifts upwards at the same time so long as  $I < W$ , but it begins to shift down when  $I$  becomes greater than  $W$ .

FIG. 7.

So long as the investment is greater than the level of maintenance of the equipment capacity  $W$ , this capacity is further expanding, thus stimulating the downward cumulative process; but after the investment  $I$  becomes lower than  $W$ , the shrinkage of the equipment begins to retard it. When the point is reached in which  $I=D$  and the investment ceases to decline, the further shrinkage of equipment causes an increase of investment decisions, and this will be the beginning of an upward cumulative process.

We have shown in the first paragraph that a cumulative process with constant equipment leads to a state in which investment decisions are equal to investment and thus the latter remains in the following  $\tau$ -periods at a constant level. Now we see that this "equilibrium" is disturbed by the change of capital equipment. After the upward cumulative process has come to an end, the rise of equipment capacity at the top of prosperity causes a downward cumulative movement, which in turn is followed by an upward process started with the contraction of capacity at the bottom of the depression. The dynamic process consists thus of a series of upward and downward cumulative processes following each other. In other words, it forms a business cycle.

3. It is useful for the understanding of the nature of the business cycle to represent it as a movement of the point in the plane  $I, D$ . In Fig. 8 we have the curves  $\phi$  representing the functional dependence of investment decisions  $D$  on investment  $I$  with various equipment capacities. The greater the equipment capacity the lower the position of the corresponding curve  $\phi$ .

We shall mark now on every curve the point whose abscissa is equal to the level  $W$  of investment needed for the maintenance of the capacity of equipment to which this curve  $\phi$  corresponds. The *locus* of all these points is the curve  $EG$ .<sup>1</sup> For all points on that curve we have  $I = W$ , for all points on the left of it  $I < W$ , whilst for all points on the right  $I > W$ .

Now if investment and investment decisions in a certain  $\tau$ -period are represented by a point  $(I, D)$  this point will move along the curve  $\phi$  towards the point of intersection with  $OL$ , while this curve will shift upwards, downwards, or remain stationary according to whether the point  $(I, D)$  lies on the left of curve  $EG$  on the right of it, or on that curve.

Let us now assume that to investment and investment decisions in the first  $\tau$ -period there corresponds the point  $E$ . In that point  $I = W$  and thus the moving point  $(I, D)$  representing the variable investment and the rate of investment decisions in our dynamic process moves along the curve  $EA$  towards  $A$  whilst this curve is stationary. Investment  $I$  increases. Because of this, however, the investment in the next period is higher than the "level of maintenance"  $W$  and the curve  $\phi$  begins to shift down. Consequently the moving point  $(I, D)$  has the trajectory  $EF$ , which is the resultant of the movement along  $\phi$  and the downward shift of it. (In point  $E$  this trajectory is tangential to the curve  $\phi$ .) In the point  $F$  the investment  $I$  ceases to grow because  $D = I$ , whilst the curve  $\phi$  shifts farther down, consequently the point  $(I, D)$  moves here vertically. In that way it falls below  $OL$ ; the investment

<sup>1</sup> This curve is descending because the lower the position of a curve  $\phi$  the greater the corresponding equipment and the greater the level of investment  $W$  by which the capacity is maintained.

decisions are now lower than investment and the latter begins to fall. The point  $(I, D)$  now moves along the curve  $\phi$  to the left, whilst this curve shifts farther down because still  $I > W$ . In that way the moving point meets the curve  $EG$  in  $G$ . The curve  $\phi$  now ceases to shift down, and the trajectory is here again tangential to  $GC$  as before to  $EA$ . But soon  $I$ , falling farther, becomes lower than  $W$  and the curve  $\phi$  begins to shift up, whilst the movement along the curve  $\phi$  is further directed to the left because still  $D < I$ . In the point  $H$  investment decisions become equal to investment, and the latter ceases to fall, while the curve shifts farther up. The point  $(I, D)$  moves here again vertically but upwards. Thus  $D$  becomes greater than  $I$ , the movement along  $\phi$  is directed to the right, while the curve  $\phi$  shifts farther up. In that way the moving point comes back to the point  $E$  and the new cycle begins.

It is clear that the moving point cannot stop at any point of the trajectory. In  $E$  and  $G$  the investment is equal to "the level of maintenance," but investment decisions are higher or lower respectively than the investment. In  $F$  and  $H$  the rate of investment decisions is equal to investment, and thus there is no tendency for a cumulative process, but investment is higher or lower respectively than the "level of maintenance" and the equipment capacity expands or shrinks. The only point in the plane  $I, D$  from which there is no tendency to move is  $B$ , the point of intersection of  $EG$  and  $OL$ . In that point  $D = I = W$ , or there is no tendency towards the cumulative process and no change of equipment capacity. It thus corresponds to long-run equilibrium. If the initial position of the moving point does not coincide with  $B$  it must move round it. In other words, if in the first  $\tau$ -period investment, investment decisions and equipment do not correspond to the point  $B$ , there must arise a business cycle.

Clearly it is an arbitrary assumption that the moving point comes back to its initial position  $E$ —the trajectory need not be a closed curve but may also be a spiral.

4. We see that the question, "What causes the periodical crisis?" could be answered shortly: the fact that the investment is not only produced but also producing. Investment considered as capitalists' spending is the source of prosperity, and every increase of it improves business and stimulates a further rise of spending for investment. But at the same time investment is

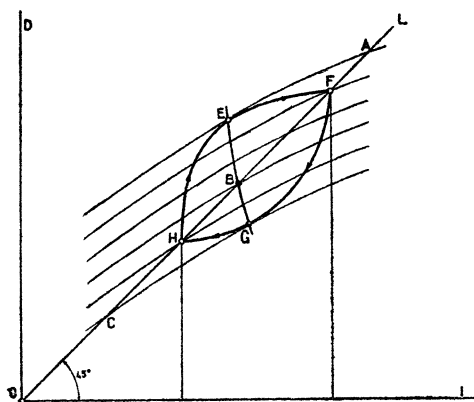


FIG. 8.

The "cracks" of the curve  $EFGH$  in  $F$  and  $H$  are due to discontinuity introduced by division of the process into  $\tau$ -periods (compare with Fig. 7: point  $F$  corresponds to period  $n$ ).

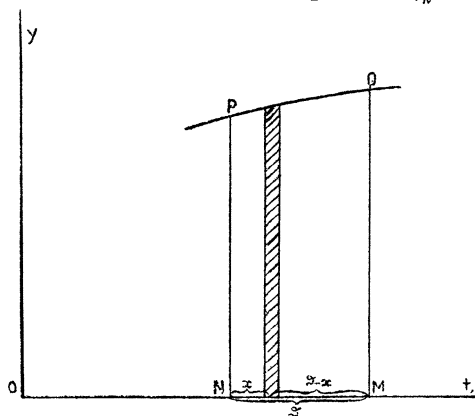
an addition to the capital equipment and right from birth it competes with the older generation of this equipment. The tragedy of investment is that it calls forth the crisis because it is useful. I do not wonder that many people consider this theory paradoxical. But it is not the theory which is paradoxical but its subject—the capitalist economy.

*London.*

MICHAL KALECKI.

## NOTE

We shall try to make tenable that the average time of finishing investment orders  $\tau$  varies only slowly within a narrow range. Let us define first  $\tau$  for one kind of investment good with gestation period  $\partial$ . In the formula  $\tau = \frac{O}{I} = \frac{\Sigma o_k}{\Sigma \tau_k^{o_k}}$  we can express  $o_k$  in terms of the investment good



which we deal with, for it is clear that in the above expression the price of this good in the given moment is irrelevant for  $\tau$ . On the chart we see the time curve  $PQ$  of the rate of investment decisions  $y$  (i.e. how much of our type of investment is ordered in a given moment per unit of time). All orders being under construction at the moment  $M$  were given in the time space  $MN$ , for all earlier orders are already completed, whilst none of these orders is finished. The order given at the moment  $x$  (reckoned from  $N$ ) is equal to  $y dx$ ; this order was under construction during a time  $\partial - x$  (because it was given  $\partial - x$  time-units ago) and thus the time needed to complete it is  $\partial - (\partial - x) = x$  and its part to be completed equals  $y dx \cdot \frac{x}{\partial}$ . Consequently,  $\Sigma o_k$  expressed in terms of investment good considered is at the moment  $M$ :

$$\omega = \frac{1}{\partial} \int_0^{\partial} y x dx$$

For  $\Sigma \tau_k^{o_k}$  expressed in terms of investment good—which is nothing else than the “real” investment  $i$  at the moment  $M$ —we get, if one takes into account that  $\tau_k$ , the time needed to complete the order  $y dx \cdot \frac{x}{\partial}$ , is here equal to  $x$ ,

$$i = \frac{1}{\partial} \int_0^{\partial} y dx$$

It follows that

$$\tau = \frac{\omega}{i} = \frac{\frac{1}{\partial} \int_0^{\partial} y x dx}{\frac{1}{\partial} \int_0^{\partial} y dx}$$

Taking approximately that  $PQ$  is a straight line segment this expression gives :

$$\tau = \frac{\partial}{2} \left( 1 + \frac{\alpha}{6} \right)$$

where  $\alpha$  is the relation  $\frac{e}{h}$  of the increase of  $y$  in the time from  $N$  to  $M$  to the average of  $y$  at  $N$  and  $M$ . Now it is clear : (1) If the rate of increase (or decrease) of  $y$  during  $\partial$  is not very great,  $\tau$  differs slightly from  $\frac{\partial}{2}$ . (2) If the rate of increase (or decrease) of  $y$  does not change much within a certain time, the change of  $\tau$  in this time is small.

Let us now come back to our general expression  $\tau = \Sigma o_k / \Sigma \frac{o_k}{\tau_k}$ . We shall divide the uncompleted projects  $o_k$  into groups each of them including all uncompleted investment orders of a certain type  $l$  with gestation period  $\partial_l$ . The "real" value of these uncompleted orders of type  $l$  is  $\omega_l$  and the correspondent  $\tau_l$ -period is equal to  $\frac{\partial_l}{2} \left( 1 + \frac{\alpha_l}{6} \right)$

Denoting the price per an investment unit at a given moment by  $p_l$ , we have now  $\Sigma o_k = \Sigma \omega_l p_l$  and

$$I = \Sigma \frac{o_k}{\tau_k} = \Sigma \omega_l p_l$$

Thus we obtain :

$$\tau = \frac{\Sigma \omega_l p_l}{\Sigma \frac{\omega_l p_l}{\frac{\partial_l}{2} \left( 1 + \frac{\alpha_l}{6} \right)}}$$

Now it is clear that if : (1) the rates of increase  $\alpha_l$  of the singular types of investment do not change much within a certain time ; (2) the distribution of the value of uncompleted investment projects among singular investment types does not change much, too—the change of  $\tau$  during this time is small. It is also obvious that if  $\alpha_l$  are not very great,  $\tau$  differs slightly from the half average gestation period.

