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The Review of Economic Studies, Vol. 21, No. 2. (1953 - 1954), pp. 81-106.

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The Production Function and the Theory of Capital

INTRODUCTION

The dominance in neo-classical economic teaching of the concept of a production function, in which the relative prices of the factors of production are exhibited as a function of the ratio in which they are employed in a given state of technical knowledge, has had an enervating effect upon the development of the subject, for by concentrating upon the question of the proportions of factors it has distracted attention from the more difficult but more rewarding questions of the influences governing the supplies of the factors and of the causes and consequences of changes in technical knowledge.

Moreover, the production function has been a powerful instrument of mis-education. The student of economic theory is taught to write $O = f(L, C)$ where L is a quantity of labour, C a quantity of capital and O a rate of output of commodities.¹ He is instructed to assume all workers alike, and to measure L in man-hours of labour ; he is told something about the index-number problem involved in choosing a unit of output ; and then he is hurried on to the next question, in the hope that he will forget to ask in what units C is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next.

The question is certainly not an easy one to answer. The capital in existence at any moment may be treated simply as " part of the environment in which labour works."² We then have a production function in terms of labour alone. This is the right procedure for the short period within which the supply of concrete capital goods does not alter, but outside the short period it is a very weak line to take, for it means that we cannot distinguish a change in the stock of capital (which can be made over the long run by accumulation) from a change in the weather (an act of God).

We may look upon a stock of capital as the specific list of all the goods in existence at any moment (including work-in-progress in the pipe lines of production). But this again is of no use outside the strict bounds of the short period, for any change in the ratio of capital to labour involves a re-organisation of methods of production and requires a change in the shapes, sizes and specifications of many or all the goods appearing in the original list.³

As soon as we leave the short period, however, a host of difficulties appear. Should capital be valued according to its future earning power or its past costs ?

When we know the future expected rate of output associated with a certain capital good, and expected future prices and costs, then, if we are given a rate of interest, we can value the capital good as a discounted stream of future profit which it will earn. But to do so, we have to begin by taking the rate of interest as given, whereas the main purpose of the production function is to show how wages and the rate of interest (regarded as the wages of capital) are determined by technical conditions and the factor ratio.

¹ Throughout this essay we shall be abstracting from land as a factor of production, so we will not bother the student with it.

² Keynes, *General Theory*, p. 214.

³ In Professor Robertson's example, when a tenth man joins nine who are digging a hole, nine more expensive spades are turned into nine cheaper spades and a bucket to fetch beer. (*Economic Fragments*, p. 47.)

Are we then to value capital goods by their cost of production? Clearly money cost of production is neither here nor there unless we can specify the purchasing power of money, but we may cost the capital goods in terms of wage units, that is, in effect, to measure their cost in terms of a unit of standard labour.

To treat capital as a quantity of labour time expended in the past is congenial to the production-function point of view, for it corresponds to the essential nature of capital regarded as a factor of production. Investment consists, in essence, in employing labour now in a way which will yield its fruits in the future while saving is making current products available for the workers to consume in the meantime; and the productiveness of capital consists in the fact that a unit of labour that was expended at a certain time in the past is more valuable to-day than a unit expended to-day, because its fruits are already ripe.

But here we encounter a fundamental difficulty which lies at the root of the whole problem of capital. A unit of labour is never expended in a pure form. All work is done with the assistance of goods of some kind or another. When Adam delved and Eve span there were evidently a spade and a spindle already in existence. The cost of capital includes the cost of capital goods, and since they must be constructed before they can be used, part of the cost of capital is interest over the period of time between the moment when work was done in constructing capital goods and the time when they are producing a stream of output. This is not just a consequence of capitalism, for equally in a socialist society a unit of labour, expended to-day, which will yield a product in five years' time, is not the same thing as a unit which will yield a product to-morrow.

Finally, even if it were possible to measure capital simply in terms of labour time, we still should not have answered the question: of what units is C composed? When we are discussing accumulation, it is natural to think of capital as measured in terms of product. The process of accumulation consists in refraining from consuming current output in order to add to the stock of wealth. But when we consider what addition to productive resources a given amount of accumulation makes, we must measure capital in labour units, for the addition to the stock of productive equipment made by adding an increment of capital depends upon how much work is done in constructing it, not upon the cost, in terms of final product, of an hour's labour. Thus, as we move from one point on a production function to another, measuring capital in terms of product, we have to know the product-wage rate in order to see the effect upon production of changing the ratio of capital to labour. Or if we measure in labour units, we have to know the product-wage in order to see how much accumulation would be required to produce a given increment of capital. But the wage rate alters with the ratio of the factors: one symbol, C , cannot stand both for a quantity of product and a quantity of labour time.

All the same, the problem which the production function professes to analyse, although it has been too much puffed up by the attention paid to it, is a genuine problem. To-day, in country Alpha, a length of roadway is being cleared by a few men with bulldozers; in Beta a road (of near-enough the same quality) is being made by some hundreds of men with picks and ox-carts. In Gamma thousands of men are working with wooden shovels and little baskets to remove the soil. When all possible allowances have been made for differences in national character and climate, and for differences in the state of knowledge, it seems pretty clear that the main reason for this state of affairs is that capital in some sense is more plentiful in Alpha than in Gamma. Looked at from the point of view of an individual capitalist, it would not pay to use Alpha methods in Gamma (even if unlimited finance were available) at the rate of interest which is ruling, and looked at from the point of view of society, it

would need a prodigious effort of accumulation to raise all the labour available in Gamma even to the Beta level of technique. The problem is a real one. We cannot abandon the production function without an effort to rescue the element of common-sense that has been entangled in it.

THE QUANTITY OF CAPITAL

“Capital” is not what capital is called, it is what its name is called. The capital goods in existence at a moment of time are all the goods in existence at that moment. It is not all the things in existence. It includes neither a rubbish heap nor Mont Blanc. The characteristic by which “goods” are specified is that they have value, that is purchasing power over each other. Thus, in country Alpha an empty petrol tin is not a “good,” whereas in Gamma where old tins are a source of valuable industrial raw material, it is.

The list of goods is quite specific. It is so many actual particular objects, called blast furnaces, overcoats, etc., etc. Goods grouped under the same name differ from each other in the details of their physical specifications and these must not be overlooked. Differences in their ages are also important. A blast furnace twenty years old is not equivalent to a brand new one of the same specification in other respects, nor is an egg twenty days old equivalent to a brand new one. There is another relevant characteristic of the goods. An overcoat requires one body to wear it, and an egg one mouth to eat it. Without one body, or one mouth, they are useless, and two bodies or mouths (at a given moment of time) cannot share in using them. But a blast furnace can be used by a certain range of numbers of bodies to turn iron ore into iron. Therefore the description of a blast furnace includes an account of its rate of output as a function of the number of bodies operating it. (Since we shall not discuss short-period problems, the number of bodies actually working each piece of equipment, in the situations with which we shall be concerned, is the number which is technically most appropriate to it.)

There is another aspect of the goods which is quite different. Of two overcoats, completely similar in all the above respects, one is on the body of Mrs. Jones, who is purring with inward delight at her fine appearance. Another is on the body of Mrs. Snooks, who is grizzling because, her husband’s income being what it is, she is obliged to buy mass-produced clothes. In what follows we shall not discuss this aspect of goods at all. We take it that an overcoat (Mark IV) is an overcoat (Mark IV), and no nonsense.

Now, this enormous who’s who of individual goods is not a thing what we can handle at all easily. To express it as a *quantity* of goods we have to evaluate the items of which it is composed. We can evaluate the goods in terms of the real cost of producing them—that is, the work and the formerly existing goods required to make them, or in terms of their value expressed in some unit of purchasing power, or we can evaluate them according to their productivity—that is, what the stock of goods will become in the future if work is done in conjunction with it.

In a position of equilibrium all three evaluations yield equivalent results ; there is a quantity which can be translated from one number to another by changing the unit. This is the definition of equilibrium. It entails that there have been no events over the relevant period of past time which have disturbed the relation between the various valuations of a given stock of goods, and that the human beings in the situation are expecting the future to be just like the past—entirely devoid of such disturbing events. Then the rate of profit ruling to-day is the rate which was expected to rule to-day when the decision to invest in any capital good now extant was made, and the expected future receipts, capitalised at the current rate of profit, are equal to the cost of the capital goods which are expected to produce them.

When an unexpected event occurs, the three ways of evaluating the stock of goods part company and no amount of juggling with units will bring them together again.

We are accustomed to talk of the rate of profit on capital earned by a business as though profits and capital were both sums of money. Capital when it consists of as yet uninvested finance is a sum of money, and the net receipts of a business are sums of money. But the two never co-exist in time. While the capital is a sum of money, the profits are not yet being earned. When the profits (quasi-rents) are being earned, the capital has ceased to be money and become a plant. All sorts of things may happen which cause the value of the plant to diverge from its original cost. When an event has occurred, say, a fall in prices, which was not foreseen when investment in the plant was made, how do we regard the capital represented by the plant?

The man of deeds, who has decisions to make, is considering how future prospects have altered. He is concerned with new finance or accrued amortisation funds, which he must decide how to use. He cannot do anything about the plant (unless the situation is so desperate that he decides to scrap it). He is not particularly interested (except when he has to make out a case before a Royal Commission) in how the man of words, who is measuring capital, chooses to value the plant.¹

The man of words has a wide choice of possible methods of evaluation but none of them is very satisfactory. First, capital may be conceived of as consisting either in the cost or in the value of the plant. If cost is the measure, should money cost actually incurred be reckoned? It is only of historical interest, for the purchasing power of money has since changed. Is the money cost to be deflated? Then by what index? Or is capital to be measured at current replacement cost? The situation may be such that no one in his senses would build a plant like this one if he were to build now. Replacement cost may be purely academic. But even if the plant is, in fact, due to be replaced by a replica of itself at some future date, we still have to ask what proportion of the value of a brand new plant is represented by this elderly plant? And the answer to that question involves future earnings, not cost alone.

If the capital is to be measured by value, how decide what the present value of the plant is? The price at which it could be sold as an integral whole has not much significance, as the market for such transactions is narrow. To take its price on the Stock Exchange (if it is quoted) is to go before a tribunal whose credentials are dubious. If the capital-measurer makes his own judgment, he takes what he regards as likely to be the future earnings of the plant and discounts them at what he regards as the right rate of interest for the purpose, thus triumphantly showing that the most probable rate of profit on the capital invested in the plant is equal to the most appropriate rate of interest.

All these puzzles arise because there is a gap in time between investing money capital and receiving money profits, and in that gap events may occur which alter the value of money.

To abstract from uncertainty means to postulate that no such events occur, so that the *ex ante* expectations which govern the actions of the man of deeds are never out of gear with the *ex post* experience which governs the pronouncements of the man of words, and to say that equilibrium obtains is to say that no such events have occurred for some time, or are thought liable to occur in the future.

The ambiguity of the conception of a quantity of capital is connected with a profound methodological error, which makes the major part of neo-classical doctrine spurious.

1

"A man of words but not of deeds
Is like a garden full of weeds."

This is sadly true of the theory of capital.

The neo-classical economist thinks of a position of equilibrium as a position towards which an economy is tending to move as time goes by. But it is impossible for a system to *get into* a position of equilibrium, for the very nature of equilibrium is that the system is already in it, and has been in it for a certain length of past time.

Time is unlike space in two very striking respects. In space, bodies moving from *A* to *B* may pass bodies moving from *B* to *A*, but in time the strictest possible rule of one-way traffic is always in force. And in space the distance from *A* to *B* is of the same order of magnitude (whatever allowance you like to make for the Trade Winds) as the distance from *B* to *A* ; but in time the distance from to-day to to-morrow is twenty-four hours, while the distance from to-day to yesterday is infinite, as the poets have often remarked. Therefore a space metaphor applied to time is a very tricky knife to handle, and the concept of equilibrium often cuts the arm that wields it.

When an event has occurred we are thrown back upon the who's who of goods in existence, and the "quantity of capital" ceases to have any other meaning. Then only that part of the theory of value which treats of the short period, in which the physical stock of capital equipment is given, has any application.

Nevertheless, some of the internal relations between the parts of a system can be most easily thought about by imagining it to be in equilibrium, and an examination of these relations is useful, provided that it is conducted with due regard to its limitations.

In what follows we are concerned with such internal relations, shown by the properties of an equilibrium situation—in particular with the interrelations between three of its properties—the quantity of capital, the labour force, and the state of technical knowledge.

ASSUMPTIONS AND DEFINITIONS

We must make certain drastic assumptions in order to isolate our problem.

I

(1) Labour is perfectly homogeneous. All men are alike, and each (when employed) performs a regular number of hours' work over a year.

(2) Land, including all non-produced means of production, is homogeneous. This involves that there are no specialised factors of production such as particular kinds of soil or mineral deposits, and no influence of geography upon production.

(3) All households consume commodities in the same proportions, irrespective of changes in their relative prices ; differences in average income per head and in the distribution of income between individuals have no effect upon the composition of demand for final output. We can then measure output simply in units of a composite commodity, representing each good in the proportion in which it is being produced. In so far as net investment is going on, capital goods are represented in the unit of final output.¹

(4) There are no economies or diseconomies of scale for output as a whole or for particular commodities.

I call these assumptions the trick assumptions, because they are only a scarecrow to keep the index number birds off our fields until after the harvest.

¹ There is a certain awkwardness in assuming that the proportion of net investment in total income is independent of the distribution of income, but this difficulty does not impinge upon the questions that we have to discuss.

II

The argument is confined to a two-factor economy, and a two-class society.

(1) To isolate the problem of capital we abstract from land and all non-produced means of production. The "free gifts of nature" are completely free, plentiful and unappropriated. Space and air are necessary to production, but neither commands a price.

(2) We rule out assets, such as the goodwill of a business, which are wealth to an individual but not to society; we mean by capital physical productive resources, which would have the same significance in an artisan, a capitalist and a socialist economy.

(3) To eliminate the influence of the entrepreneur, we assume that economies of scale internal to a productive unit are exhausted, in each line of production, at a moderate rate of output, and that an experienced entrepreneur has no advantage over a new hand. The know-how of production is widely diffused and differences in skill of management are unimportant. The only qualification required for employing labour is then the ownership of sufficient capital (or command of sufficient credit) to set up a productive unit of the minimum size. (The assumption of no economies of scale (I, 2) appears here as well as in the scarecrow.) In these conditions, the distinction between interest and profit ceases to be significant. There is no specific "reward of enterprise" apart from the reward of owning capital, and no owner of capital will provide finance for an entrepreneur at appreciably less than the rate of profit which he can expect to obtain by using his capital himself to employ labour. Thus in our system, profits and wages exhaust total net income.

The size of individual productive units, above the technical minimum, is not strictly limited in the long run, but there is no particular pressure towards a growth in size, and, when the total amount of capital is accumulating, the number of independent capitalists is conceived to multiply more or less in proportion to the amount of capital; the average scale on which they operate is more or less constant, at a size which is small in relation to the markets supplied, so that conditions of atomistic competition can prevail. (The minimum size of a productive unit is, however, large enough to make it very difficult for a worker to become a capitalist, so that the system does not relapse into an economy of artisans.)

III

We call the stock of goods in existence at any moment *physical capital*. The value of these goods in terms of a unit of output we call *capital simpliciter*. Capital valued in terms of wage units we call *real capital*; though it must be observed that there is a slightly misleading flavour about this term, since the cost of capital goods, in terms of wage units, includes interest over the time required to construct them and to use them in production. Thus the same stock of physical goods represents a larger amount of *real capital* when the rate of interest is higher (and has been higher in the past) than when it is (and has been) lower.

We call the ratio of real capital to man hours of current employment per annum the *factor ratio*.

We take as the wage unit the price of an hour's labour in terms of the composite unit of product, no matter whether the worker who performs an hour's work is paid in cash or in peanuts.¹ In what follows we mean by "wages," the cost of labour to

¹ When Mr. (now Professor) Hicks eliminated the equation for money from the $n + 1$ value equations for n commodities, Mr. Lerner remarked, "If I eliminate the equation for peanuts, what then?" I take peanuts as an example of a commodity chosen at random, in allusion to this extremely sapient contribution to the pure theory of value.

the employer, in terms of product. When we have occasion to relax the trick assumptions and to look at wages from the point of view of workers regarded as consumers, we call the purchasing power of the wage a worker gets his real wage. Thus we are using the "real wage rate" in its common or garden sense, and the "wage rate" simpliciter in a special sense.

LONG-PERIOD EQUILIBRIUM

Our argument treats of the relations between quantities of factors of production in existence. It cannot take into its purview the disturbances arising from the process of changing the quantities of factors. We must, therefore, rule out all problems of effective demand and confine our argument to positions of equilibrium. What does this imply?

One notion of equilibrium is that it is reached (with a constant labour force) when the stock of capital and the rate of profit are such that there is no motive for further accumulation. This is associated with the idea of an ultimate thorough-going stationary state,¹ in which the rate of profit is equal to the "supply price of waiting." In this situation an accidental increase in the stock of capital above the equilibrium quantity would depress the rate of profit below this supply price, and cause the additional capital to be consumed; while any reduction would raise the rate of profit, and cause the deficiency to be made good. Equilibrium prevails when the stock of capital is such that the rate of profit is equal to the supply price of that quantity of capital.

But this notion is a very treacherous one. Why should the supply price of waiting be assumed positive? In Adam Smith's forest there was no property in capital and no profit (the means of production, wild deer and beavers, were plentiful and unappropriated). But there might still be waiting and interest. Suppose that some hunters wish to consume more than their kill, and others wish to carry consuming power into the future. Then the latter could lend to the former to-day, out of to-day's catch, against a promise of repayment in the future. The rate of interest (excess of repayment over original loan) would settle at the level which equated supply and demand for loans. Whether it was positive or negative would depend upon whether spendthrifts or prudent family men happened to predominate in the community. There is no *a priori* presumption in favour of a positive rate. Thus the rate of interest cannot be accounted for as the "cost of waiting".

The reason why there is always a demand for loans at a positive rate of interest, in an economy where there is property in the means of production and means of production are scarce, is that finance expended now can be used to employ labour in productive processes which will yield a surplus in the future over costs of production. Interest is positive because profits are positive (though at the same time the cost and difficulty of obtaining finance play a part in keeping productive equipment scarce, and so contribute to maintaining the level of profits).

Where the "supply price of waiting" is very low or negative, the ultimate stationary equilibrium cannot be reached until the rate of profit has fallen equally low, capital has ceased to be scarce and capitalism has ceased to be capitalism. Therefore this type of equilibrium is not worth discussing.

The other way of approaching the question is simply to postulate that the stock of capital in existence at any moment is the amount that has been accumulated up to date, and that the reason why it is not larger is that it takes time to grow. This is the conception which is adopted in this essay. At any moment there is a certain stock of capital in existence. If the rate of profit and the desire to own more wealth are such

¹ Pigou, *The Economics of Stationary States*.

as to induce accumulation, the stock of capital is growing and, provided that labour is available or population growing, the system is in process of expanding without any disturbance to the conditions of equilibrium. (If two snapshots were taken of the economy at two different dates, the stock of capital, the amount of employment and the rate of output would all be larger, in the second photograph, by a certain percentage, but there would be no other difference.) If the stock of capital is being kept constant over time, that is merely a special case in which the rate of accumulation happens to be zero. (The two snapshots would then be indistinguishable.)

In the internal structure of the economy conditions of long-period equilibrium are assumed to prevail. Each type of product sells at its normal long-run supply price. For any one type of commodity, profit, at the rate ruling in the system as a whole, on the cost of capital equipment engaged in producing it, is part of the long-run supply price of the commodity, for no commodity will continue to be produced unless capital invested for the purpose of producing it yields at least the same rate of profit as the rest. (It is assumed that capitalists are free to move from one line of production to another.) Thus the "costs of production" which determine supply price consist of wages and profits. In this context the notion of a quantity of capital presents no difficulty, for, to any one capitalist, capital is a quantity of value, or generalised purchasing power, and under our trick assumptions, in a given equilibrium situation, a unit of any commodity can be used as a measure of purchasing power.

Since the system is in equilibrium in all its parts, the ruling rate of profit is being obtained on capital which is being used to produce capital goods, and enters into their "cost of production". Profit on that part of the cost of capital represented by this profit is then a component of the "cost of production" of final output. A capitalist who buys a machine ready made pays a price for it which includes profit to the capitalist who sells it. The profit a capitalist who has the machine built in his own workshops will expect to receive, from sales of the final output, includes profit on the interest (at a notional rate equal to the ruling rate of profit) on the cost of having the machine built reckoned over the period of construction. For when he builds the machine himself he has a longer waiting period between starting to invest and receiving the first profit. If he could not earn profit on the notional interest cost, he would prefer to make an investment where there was a shorter waiting period, so that he could receive actual profit earlier. The actual profit he could plough into investment; thus acquiring (over the same waiting period) the same quantity of capital as in the case where he builds the machine for himself. (He would also have the advantage that he could change his mind and consume the profit, whereas in the first case he is committed to the whole scheme of investment once he begins.) Thus investments with a long gestation period will not be made unless they are expected to yield a profit on the element of capital cost represented by compound interest over the gestation period (if there were uncertainty, they would have to be expected to yield more, to compensate for the greater rigidity of the investment plan).

We need not go back to Adam to search for the first pure unit of labour that contributed to the construction of existing equipment. The capital goods in being to-day have mutually contributed to producing each other, and each is assumed to have received the appropriate amount of profit for doing so.

So much for the supply price of an item of new equipment. How are we to reckon the supply price of part-worn equipment? Investment in new equipment is not made unless its gross earnings (excess of output over wages bill in terms of output) are expected to be sufficient to amortise the investment over its working life, allowing for interest at the ruling rate on accrued amortisation funds, as well as providing profit at the ruling rate. The supply price of an equipment which has been working for a

certain time may be regarded as its initial cost accumulated up to date at compound interest, *minus* its gross earnings also accumulated from the dates at which they accrued up to the present, for this corresponds to the expectations which induced capitalists in the past to make the investment concerned.

Since initial cost is incurred at the beginning, and earnings accrue over time, the element of interest on cost in the above calculation exceeds the element of interest on earnings. Thus when an equipment has yielded a quarter of its expected total earnings, its supply price, in this sense, is somewhat more than three-quarters of its initial cost; half-way through, somewhat more than half its initial cost, and so forth, the difference at any moment being larger the higher the rate of interest. Over its life the accumulated interest on its earnings, so to say, catches up upon the accumulated interest on its cost, so that at the end of its life it is fully paid off and its supply price (abstracting from scrap value) has fallen to zero.

The value of an equipment depends upon its expected future earnings. It may be regarded as future earnings discounted back to the present at a rate corresponding to the ruling rate of interest. In equilibrium conditions the supply price (in the above sense) and the value of an equipment are equal at all stages of its life.¹

Equilibrium requires that the stock of items of equipment operated by all the capitalists producing a particular commodity is continuously being maintained. This entails that the age composition of the stock of equipment is such that the amortisation funds provided by the stock as a whole are being continuously spent on replacements. When the stock of equipment is in balance there is no need to enquire whether a particular worker is occupied in producing final output or in replacing plant. The whole of a given labour force is producing a stream of final output and at the same time maintaining the stock of equipment for future production. Nor is it necessary to inquire what book-keeping methods are used in reckoning amortisation quotas. These affect the relations between individual capitalists, but cancel out for the group as a whole.

In equilibrium the age composition of the stock of equipment is stable, but the total stock may be in course of expanding. The average age of the plants making up a balanced stock of stable age composition varies with the length of life of individual plants. If the total stock is remaining constant over time, the average age is equal to half the length of life. If the stock has been growing the proportion of younger plants is greater and average age is less than half the life span. (There is an exact analogy with the age composition of a stable population.)

The amount of capital embodied in a stock of equipment is the sum of the supply prices (reckoned as above) of the plants of which it is composed, and the ratio of the amount of capital to the sum of the costs of the plants when each was brand new is higher the greater the rate of interest.²

¹ The equalisation of the value of two annuities at any point of time entails their equalisation at any other point of time. If the cost of a new machine is equal, at the moment when it is brand new, to the discounted value of its expected gross earnings, it follows that, at any later point of time, the accumulated value of the original cost and gross earnings up to date will, if expectations have been proved correct up to date and are unaffected for the future, be equal to the present value of the remaining gross earnings expected over the future. Cf. Wicksell, "Real Capital and Interest," *Lectures* (English edition), Vol. I, p. 276.

² The order of magnitude of the influence of the rate of interest is shown by the formula provided in the Mathematical Addendum by D. G. Champernowne and R. F. Kahn. For this formula it is necessary to assume (a) that the total stock of capital is constant over time, (b) that earnings are at an even rate over the life of the plant. C is the capital value of an investment, K the initial outlay, r the rate of interest and T the period over which the asset earns. For values of rT less than 2 we use the approximation $C/K = \frac{1}{2} (1 + \frac{1}{2} rT)$.

On this basis, when the rate of interest is, for example, 6 per cent, a machine of ten years' life costing £100 when new must earn £13.3 per annum surplus over the current outlay on working it (including current repairs). The yield will then be 6 per cent on a capital value of £55.

(Continued overleaf.)

Equilibrium requires that the rate of profit ruling to-day was expected to be ruling to-day when investment in any plant now extant was made, and the expectation of future profits obtaining to-day was expected to obtain to-day. Thus the value of capital in existence to-day is equal to its supply price calculated in this manner. The heavy weight which this method of valuing capital puts upon the assumptions of equilibrium emphasises the impossibility of valuing capital in an uncertain world. In a world where unexpected events occur which alter values, the points of view of the man of deeds, making investment decisions about the future, and of the man of words making observations about the past, are irreconcilable, and all we can do is botch up some conventional method of measuring capital that will satisfy neither of them.

THE TECHNIQUE OF PRODUCTION

How can we reduce the amorphous conception of a "state of technical knowledge" to definite terms? Let us suppose that for any given line of production, we can draw up a list of actual techniques which could be used, with a given amount of current labour, to produce a flow of output of the commodity concerned, while maintaining the productive equipment required intact. Each technique is conceived to be specified in detail, and entails the use of particular items of equipment and a particular quantity of work-in-progress in the productive pipe line. Other things equal, a technique involving a longer production period (from clipping the sheep to selling the overcoat) requires a larger run-out of man hours embodied in work-in-progress. This is treated as part of the stock of capital goods required by this technique.¹ We then amalgamate the lists for particular commodities in such a way as to get a flow of output of commodities in the proportions dictated by the assumption that the composition of final output is given. We thus have a set of blue prints of techniques, each of which could be used to employ a given amount of labour to produce a flow of output.

The techniques are listed in a hierarchy, Alpha, Beta, etc., according to the rate of output which they produce with a given number of men. (The number of men must be a common multiple of the numbers required by a self-contained unit of each technique, to avoid "a ragged edge" when workers are allotted to plants.)

The internal description of a given technique is a purely engineering question, but the list of techniques cannot be drawn up in purely engineering terms, without

A capitalist who operates such a machine may amortise the initial investment by paying £10 every year into a sinking fund. Reckoning at simple interest only, he receives interest on the amortisation fund after one year of £0.6; in the sixth year £3; in the last year £5.4. Thus the annual return on his investment of £100 rises over the ten year period from approximately £3 to approximately £9 (the "ragged edge" is due to reckoning the amortisation quota as paid at the end of a year, and reckoning interest as paid annually instead of continuously). The undiscounted average annual income is, therefore, £6 (6 per cent on the initial outlay). Compound interest over the period compensates for discounting this income. Over the life history of the machine, the fall in capital value from £100 to zero is in step with the rise in the amortisation fund.

A group of ten such machines of ages zero to nine years have a pattern of values, at any moment, which corresponds to the pattern over time of a single machine. It requires an annual outlay on renewals of £100 permanently to maintain the stock of machines. They represent a capital value of £550 and yield a return of £33 per annum.

If the rate of interest were 10 per cent, rT would be equal to 1 and the capital value (abstracting from a higher initial cost of machines due to the higher interest rate) would be £583; the earnings of each machine would then have to be £15.8 to yield the required rate of profit.

If the length of life of machines was twenty years, and the rate of interest 5 per cent, capital value would again be £583, and each machine would have to yield £7.9 per annum (£5 for amortisation and £2.9 for interest); at 10 per cent, rT would be equal to 2; the capital value would then be £666, and each machine would have to yield £11.7 per annum.

¹ This way of looking at things is easier to understand than applying the notion of the "length of the period of production" to long-lived equipment. It is hard to treat, say, a loom as a length of time, but perfectly easy to regard a quantity of wool, a quantity of yarn and a quantity of cloth as part of the physical equipment required for producing a steady flow of output of overcoats.

regard to economic considerations. (From an engineering point of view, it is possible to use a steam hammer for cracking nuts.) We must therefore compare the costs of the equipments required by various techniques in order to be sure that we are nowhere using more capital to produce less product.¹

The cost of an equipment can be reckoned in wage units (that is, labour time), but it includes an allowance for interest on costs incurred in the past to create the stock of goods in existence to-day (this, as we have seen, depends partly upon the time taken to construct capital goods and partly upon the time over which they are used). Thus cost in wage units must be expressed as a function of the rate of interest. Let us imagine that we proceed by first taking any reasonable value for the rate of interest for a preliminary run over the field. Take an outfit of equipment as a going concern, with all its pipe lines full of work-in-progress, composed of plants of the age distribution appropriate to the rate (which may be zero) at which the total stock of capital is expanding. Now imagine that our notional rate of interest represents the rate of profit on capital actually ruling to-day, and that the same rate of profit has been ruling as long as any item in the stock of capital goods now extant has been in existence. Then we reckon the supply price of the equipment at that rate of interest, assuming that each item in the stock has been earning profits at that rate since it came into service.

When the equipment required by the techniques has been costed in this way, any technique which involves a greater cost than another for the same or a smaller rate of output is ruled out, for it is uneconomic (at the assumed rate of interest), however beautiful it may be from an engineering point of view. Then an Alpha technique produces a higher rate of output with a given amount of labour, and involves a greater cost of equipment, than a Beta technique, Beta a higher output and greater cost of equipment than Gamma, and so on down the hierarchy. We shall describe the Alpha technique as "more mechanised" than the Beta technique, and so on down the list, for Alpha involves a greater quantity of real capital (capital in terms of past labour invested) per unit of labour currently employed, than does Beta, and this will normally show itself in a greater complexity of the equipment used. In so far as the advantage of the superior technique lies in a longer life for items of equipment, it shows itself in a smaller proportion of the given labour force being occupied at any moment in replacement of capital goods. The concept of "mechanisation" is also stretched to cover working capital. Thus, in the famous example of the wine cellar, a set of barrels one of every age from one to ten years is regarded as equipment for a more mechanised technique than one consisting of barrels of ages from one to nine years (each cellar being tended by one night watchman). But this example is sophisticated by the fact that the output of the "more mechanised" technique is superior in quality (the age of the wine representing final output), not quantity, to that of the "less mechanised."

We now repeat the costings at all rates of interest over a reasonable range. In the course of this process we may create gaps in the list of techniques, or find gaps formerly existing filled up, as the notional rate of interest alters. For example, if the man-hours required to construct a plant appropriate to Gamma technique are spread over a long time, or are heavily concentrated at the beginning of the gestation period, while those required to construct a Beta plant are spread over a short time or are bunched near the moment of completion, or if a Gamma plant is more durable, so that the average age of the items making up a balanced outfit of plants is greater, a rise in the rate of interest may raise Gamma's cost above Beta's. But Gamma's rate

¹ This seems to have been the point that Keynes had in mind when he compared the lengthiness of a productive process to its smelliness. (*General Theory*, p. 215.)

of output is lower. Thus at this notional rate of interest Gamma falls out of the hierarchy.¹

Techniques may appear or disappear in the list as the notional rate of interest alters, but two techniques can never reverse their positions, for they were listed in the first place in order of rates of output with a given amount of current labour, and this is a purely engineering fact, independent of the rate of interest.

The difference between a more and less mechanised technique is not produced by adding some spoonfuls of investment to a *pot-au-feu* of "capital". Each technique involves its own specific blue prints, and there may be no recognisable items in common between one and any other. There is, therefore, no reason why the hierarchy should consist of small steps in output per man. It may do so, or it may consist of a series of jumps with appreciable gaps between each technique and the next. It seems obvious, for instance, that large jumps occur between techniques involving different sources of power.

The individual capitalist is assumed to choose between possible techniques in such a way as to maximise the surplus of output that a given amount of capital yields over wages cost in terms of his own product, and thus to obtain the highest rate of profit on capital that the available techniques make possible.²

Given the hierarchy of techniques, the higher is the wage rate the more mechanised is the technique which is chosen. This principle is usually described in a somewhat mystifying way in terms of a "substitution of capital for labour" as the cost of labour rises. The essential point, however, is very simple. An Alpha plant involves a greater capital cost and yields a higher rate of output with a given amount of current labour than a Beta plant. At a higher wage rate both plants yield a smaller profit per man employed than at a lower wage rate, but a given difference in the wage rate reduces the excess of output over wages (that is, profit) in a smaller proportion where output is higher.

This can be illustrated by means of a crude numerical example, comparing the profitability of three techniques at two wage rates. To keep the arithmetic simple we take very large differences between techniques. The difference in the capital cost of the same plant at two different wage rates is not proportional to the difference in the wage rate, for at a higher wage there is a lower rate of profit prevailing and consequently a smaller element of interest in cost. Again to keep the example simple, we assume that the capital costs of the three plants are affected in the same way by a difference in the rate of interest, so that their relative costs are the same at both wage rates.

At the wage rate of 1 per man, Gamma and Beta techniques yield the same rate of profit, and Alpha technique a lower rate. At the wage 1.1, Beta and Alpha yield the same rate of profit, and Gamma a lower rate (in fact, zero). Thus when wages are one unit of product per man year, the individual capitalist is indifferent between Gamma and Beta technique—52 units of capital in terms of value purchase one Beta plant yielding a profit of 10 per annum, or two Gamma plants yielding 5 each. (If there were any uncertainty about future profits, the Gamma technique would be preferred, since an investment which is technically divisible is more flexible than one which is an integrated whole.) Alpha technique is out of the question. Similarly, when the wage rate is 1.1, Beta and Alpha are indifferent and Gamma is out of the question.

¹ An example of this phenomenon is illustrated in Fig. I.

² For simplicity of exposition we are postulating integrated production, so that raw materials, power, etc., bought by one capitalist from others do not appear as costs.

Number of Men per Plant : 50

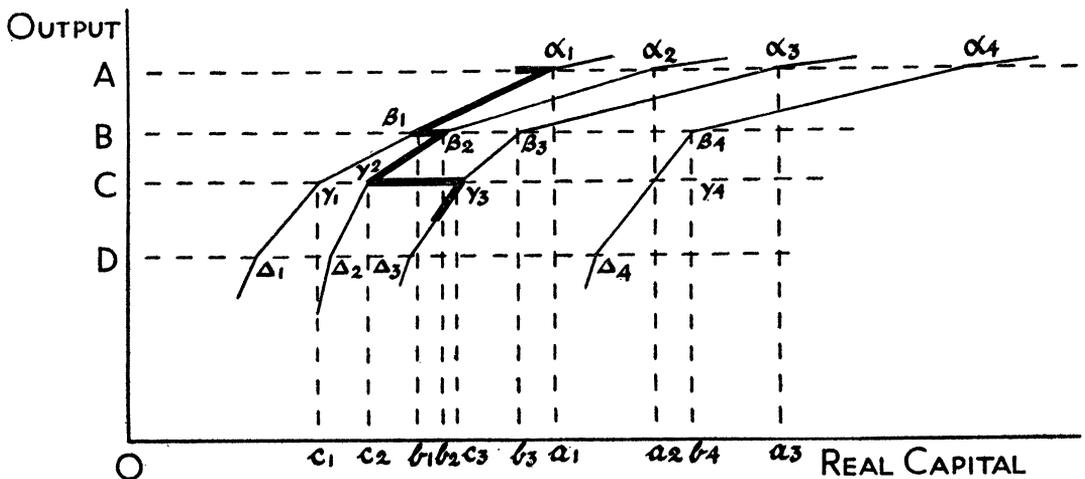
Plant	γ	β	α	γ	β	α
Wage rate ..	1	1	1	1.1	1.1	1.1
Capital ..	26	52	104	27.5	55	110
Product ..	55	60	65	55	60	65
Wage bill ..	50	50	50	55	55	55
Profit	5	10	15	0	5	10
Rate of Profit (approximate)	19%	19%	14%	0	9%	9%

In an equilibrium position the technique of production throughout the economy has been chosen according to this principle,¹ and the factor ratio (real capital per man employed), given the technical possibilities, is governed by the wage rate.

THE RATIO OF CAPITAL TO LABOUR

When the hierarchy of techniques has been specified we can draw a factor-ratio curve connecting real capital per man employed with the rate of output. The reader is warned that it has a somewhat bizarre appearance compared to the smooth sweep of the usual text-book production function.

FIG. I



OA is the rate of output of a constant number of men using Alpha technique, OB the output with Beta technique, OC with Gamma technique, and OD with Delta technique. Oa_1 is the cost in terms of wage units of a balanced outfit of equipment

¹ For a possible case of multiple equilibrium see Appendix, p. 103.

required by Alpha technique, calculated at a certain rate of interest, Oa_2 the cost of the same equipment at a higher rate of interest, and so for each of the techniques at ascending interest rates.¹

However large the jump may be between one technique and the next there is a continuous relationship between output per head and the factor ratio. Between, say, outputs OC and OB , Gamma and Beta techniques are both being employed (as in the numerical example, when the wage rate was 1) and between OB and OA , Beta and Alpha techniques are both being employed (as, in the example, at the wage rate of 1.1). A rise in the factor ratio from Oc_1 to Ob_1 , or from Oc_2 to Ob_2 , is due to a gradual increase in the proportion of Beta plants in use, which causes a rise in the average of real capital per man and a rise in average output per head.

Thus we can draw, for each rate of interest, a *productivity curve*, consisting of a series of straight lines of changing slope, which exhibits the rise of output due to ascending the hierarchy of techniques.

Each curve purports to show the engineering characteristics of the techniques, but it would be of no use to ask an engineer how they should be drawn. He does not understand the meaning of a given state of technical knowledge, for he is learning something fresh everyday as he works. "Given knowledge" is a drastic abstraction (though it may have some relevance for a "backward" country which can use a survey of past and present techniques operated in "advanced" countries as a catalogue of possibilities to choose from). It implies some absolute upper limit to the rate of output that a given labour force can produce. Our curves, therefore, must be drawn with a maximum output as an asymptote and their general shape is concave to the x axis². To avoid complicating the exposition we will postulate that they are concave throughout.³

The relation between one curve and the next depends upon the reaction of the cost of various outfits of equipment to differences in the rate of interest, and this depends, as we have seen, in a complicated way, upon the gestation period and length of life of items of equipment. There is little to be said about it *a priori*,⁴ though it is reasonable to suppose that the most mechanised techniques are the most sensitive to the rate of interest, so that the family of curves fans out laterally as it rises.⁵

The data embodied in this system of curves (if only we had any data!) provide a complete description of all the quantities of capital, valued in wage units, which can be used, in a given state of knowledge, to employ a constant labour force.

Now, the conditions of equilibrium require that the rate of interest which enters into the cost of equipment is equal to the rate of profit actually ruling (for that rate

¹ To illustrate the point made above, Gamma technique is shown as becoming uneconomic at the fourth interest rate.

² This expresses "diminishing returns to capital" as the ratio of real capital to labour rises. It is important not to confuse diminishing returns in this sense with the Classical law of diminishing returns. Classical diminishing returns arise from an increase of population relatively to constant natural resources, which may well correspond to the facts of life, whereas the diminishing returns shown in the production function are the result of the artificial assumption of a given state of knowledge.

³ There is no reason, from an engineering point of view, why they should not be convex over particular ranges, but if the slope, say, between a γ point and a β point is less than the slope between β and the corresponding α , it would indicate that the increase in output due to substituting an Alpha for a Beta plant would be more than proportionate to the increase, at a given wage rate, in the cost of equipment involved, so that Beta technique would never be profitable to use. This possibility is not ruled out by the process of eliminating uneconomic techniques from the curves (see p. 91). E.g. in Fig. 1, if γ_4 lay between the perpendicular β_4b_4 and the line $\delta_4\beta_4$, the Gamma technique would not have been eliminated, but the curve $\delta_4\gamma_4\beta_4$ would be convex, and there could be no equilibrium on this section.

⁴ The Mathematical Addendum indicates the lines on which it would be possible to work out the influence of the rate of interest on the cost of equipments having various characteristics in respect to length of life, etc. Cf. p. 89, note 2.

⁵ For a "perverse" case which may occur when this is not true see Appendix, p. 106.

of profit has been ruling over the period when the existing stock of capital goods was being constructed). We must, therefore, imagine that, by a process of trial and error, we find a position, for each factor ratio, where the two are congruent. The productivity curve, says, $\gamma_2\beta_2\alpha_2$, is drawn up on the basis of a rate of interest equal to the rate of profit which would obtain if the wage rate were such as to make Gamma and Beta techniques equally profitable. The thick line in the diagram is the factor-ratio curve.

At γ_2 the factor ratio is Oc_2 , and all men are employed with Gamma technique. An increase in the factor ratio from Oc_2 towards Ob_2 and a rise of output from OC towards OB would come about by substituting Beta for Gamma plants. When the factor ratio has risen to Ob_2 all workers are employed with Beta technique, and output is OB . A further increase in the factor ratio can come about only by the introduction of Alpha plants, but this requires a rise in the wage rate and entails a fall in the rate of profit. We, therefore, jump horizontally, from β_2 to β_1 , onto the productivity curve corresponding to the rate of profit which obtains when the wage rate is such that Alpha and Beta techniques are equally profitable. The factor ratio increases from Ob_1 to Oa_1 by the substitution of Alpha for Beta plants until, at Oa_1 all men are employed with Alpha technique. A further increase in the factor ratio then requires a rise in the wage rate. And so on, up the hierarchy of techniques, from one productivity curve to another. At the final upper limit, where a further increase in the factor ratio cannot further increase output per man, we reach the state of Bliss, where wages absorb the whole product and capital has ceased to be scarce relatively to the state of knowledge.

The foregoing analysis shows that the relation of capital to labour, in an equilibrium position, can be regarded as the resultant of the interaction of three distinct influences: the wage rate, the rate of interest and the degree of mechanisation.

The influence of the wage rate upon the value in terms of product of given physical capital was emphasised by Wicksell,¹ and has been called the *Wicksell effect*.² When we regard a stock of capital as the result of accumulation brought about by saving—that is, refraining from consuming income—we measure the saving in terms of consumption forgone, and the accumulated capital as a sum of value in terms of product. The influence of the Wicksell effect (leaving aside for the moment the influence of interest on the cost of capital goods) determines the amount of physical capital which a given amount of past accumulation has brought into existence.

This influence can be distinguished from the effect of mechanisation by considering a case in which there is only one technique known. Suppose that to employ a man requires a specific set of capital goods, which we may call for convenience a machine, though it includes work-in-progress as well as long-lived equipment. Without just this machine, a worker can produce nothing, and no other kind of equipment has ever been thought of. Then comparing two situations in one of which, Beta-one, the wage rate is higher than in the other, Beta-two, a given amount of capital in Beta-one corresponds to a smaller number of machines and provides less employment than in Beta-two. This has evidently nothing to do with a "substitution of capital for labour" for, in a technical sense, no substitution is possible.

The operation of the Wicksell effect is counteracted by the operation of the *interest effect*. A higher wage rate entails a lower rate of profit and, therefore, in equilibrium, requires that a lower rate of interest has been ruling in the past. The cost of given physical capital in terms of wage units is less the higher the wage rate.³

¹ Loc. cit., p. 292.

² C. G. Uhr, "Knut Wicksell. A Centennial Evaluation," *American Economic Review*, December, 1951.

³ This is shown in the diagram by the backward jump of the factor-ratio curve from β_2 to β_1 . See also Appendix.

It is evidently possible that the interest effect should also outweigh the Wicksell effect, so that the value of given physical capital in terms of product is smaller at a higher wage rate. This would occur if, first, the cost of capital goods in terms of wage units reacts strongly to changes in the rate of interest (their gestation period *plus* their working life is long), and second, the wage rate is already high relatively to output per man, so that a given rise in the wage rate produces a large proportionate fall in the share of profit in product, and so in the rate of profit on capital.¹ Where the interest effect more than offsets the Wicksell effect we see the apparently paradoxical result that a given amount of capital (in terms of product) provides a smaller amount of employment at a lower than at a higher wage rate.

The higher factor ratio associated in equilibrium with a higher wage rate, due to the use of more mechanised technique, has been called the "Ricardo effect."² The attribution is somewhat forced, yet a label is useful ; we may call it the *Ricardesque effect*.

A rise up the hierarchy of techniques must be associated with a rise in the wage rate. But the more capital (in terms of accumulated product) has been absorbed by increasing the amount of machinery in existence, the lower the wage rate associated with a given amount of accumulation. The more the capitalists have been able to take advantage of the Ricardesque effect, the less the workers have benefited from the Wicksell effect.

(When progress in technical knowledge is economising capital in terms of accumulation by increasing the productivity of a given amount of real capital or when opportunities for mechanising production stimulate accumulation which would not otherwise occur, the case is altered. Our equilibrium conditions tell us nothing about the effect of inventions, or the vagaries of effective demand.)

REAL WAGES

The neo-classical system is based on the postulate that, in the long run, the rate of real wages tends to be such that all available labour is employed. In spite of the atrocities that have been committed in its name there is obviously a solid core of sense in this proposition. To return to our road builders, employment per unit of output is much higher in Gamma than in Alpha, and it seems obvious that this is connected with the fact that real wages there are much lower—that the plethora of labour keeps real wages down, and so helps to get itself employed. Let us try to see what this means.

The basic data of our system are : the labour force, the amount of capital,³ and the state of technical knowledge, expressed as the hierarchy, ranged according to degrees of mechanisation, of the possible techniques of production. In order to satisfy the neo-classical postulate of full employment, the given amount of capital must employ the given amount of labour.

¹ The numerical example on p. 89, note 2, shows that, on the stated assumptions, a very large reduction in the rate of interest, from 10 per cent to 6 per cent, reduces the supply price of a given balanced outfit of equipment, when the life of individual items is ten years, only in the ratio of 58 to 55 ; with a life as long as twenty years, a reduction in the rate of interest from 10 per cent to 5 per cent reduces supply price in the ratios of 66 to 58. This suggests that the interest effect is not very large. On the more realistic assumption that costs for repairs rise with the age of plant, so that earnings are larger in the earlier years, the effect of interest would be less than in the example. The rise in the wage rate entailed by the fall in the rate of interest must in most ordinary cases lead to a rise, on balance, in the cost of capital in terms of product, and cases in which the interest effect more than offsets the Wicksell effect seem likely to be rather peculiar.

² Hayek, "The Ricardo Effect," *Economica*, May, 1942

³ Accumulation may be in course of proceeding in an equilibrium position but, if so, it is going on slowly, relatively to the amount of capital already in existence, and it is going on in a manner which does not violate the internal equilibrium of the system, so that, at any moment, it is legitimate to postulate an existing amount of capital.

At any given wage rate, the interplay of competition between capitalists, each seeking to maximise his own profits, is assumed to ensure that the technique will be chosen that maximises the rate of profit. Thus the technique is a function of the wage rate. The outfit of productive equipment in existence is determined by the technique and the total amount of capital. A given outfit of equipment offers a given amount of employment. Thus we have the amount of employment as a function of the wage rate. We can then state the neo-classical postulate : the wage rate is assumed to be such that the technique of production is such that the given quantity of capital employs the given labour force. It is necessary to postulate that the amount of real wages (which is not the same thing as the wage bill but is governed by it) in relation to the cost of subsistence is at least sufficient to maintain the given labour force in being.

There is no difficulty in principle (though infinite complication) in removing the trick assumptions and introducing into our analysis specialised natural resources, varieties of skill of labour, and demand equations for individual commodities in terms of relative prices, total income and the distribution of income between households. (We could not, however, digest economies of scale, for that implies the existence of active entrepreneurs, as opposed to our capitalists whose only function is to own capital and use it to employ labour.) We should then be able to make use of the supply-and-demand analysis which neo-classical economics has developed with so much elaboration, and to show that the technique will be chosen, each capitalist reckoning wages in terms of his own product, that maximises the total of profits.

The equation relating the amount of employment to the wage rate is independent of the medium in which wages are paid to workers. Since it is more usual to pay in money than in peanuts, let us see how it works out when wages are paid in money, that is, some unit of generalised purchasing power.

The relation of the money prices of particular commodities to their money-wage costs is influenced by supply and demand for scarce factors, by the level of effective demand and by the price policies of imperfectly competitive rival producers. At any given money wage level, the resulting complex of prices determines the wage in terms of a composite unit of the product which is actually being produced. We have ruled out fluctuations in effective demand but we must consider the influence of competition.

Competition which is relevant to our argument is competition in the long-period sense—the readiness with which capitalists break into a market where a more than average rate of profit is ruling and bid down the price of the commodity being sold there. When a market is dominated by a monopolist, price in relation to money wages is such that the rate of profit on his capital is higher than the average for the economy as a whole. Capitalists from outside are anxious to enter this market, and in doing so they cut the price of the commodity, thus raising the wage rate everywhere in terms of this bit of the composite product, while lowering it in other lines from which capital is deflected. When competition is free and active, as we assume it to be, this process of competing away excess profits and raising sub-average profits is always completely successful, and at any moment a uniform rate of profit is ruling throughout the system.

When profits are being held above the general level in a particular line, by monopolistic restrictions, the process is impeded, for the existence of monopoly (in the long-period sense here appropriate) means precisely that an individual capitalist is able to prevent others from breaking into his preserves and competing away his profits.

In this situation a trade union may succeed in raising the money wage that its members receive, so that wages in terms of product in general for that particular group of workers are raised, while the monopolist (pursuing some long-range policy) does not necessarily counter by raising the money price of his commodity corres-

pondingly, so that the wage of labour taken as a whole is raised. The existence of a few monopolies here and there does not necessarily depress real wages for labour as a whole ; indeed it may cause them to be higher than that which corresponds to the competitive equilibrium position, for the monopolists may be exploiting other capitalists and sharing the spoil with their own workers. But the general prevalence of monopoly must depress real wages (unless the strength of the trade unions and the complaisance of the monopolists are so great as to prevent the rate of profit from rising above the competitive level) for by keeping up money prices it reduces the wage rate, in our sense, which governs the real wage rate that interests the workers in their capacity as consumers.

When there are elements of monopoly in an economy, different rates of profit obtain in different lines of production, without much tendency to equalisation throughout the system. The argument in terms of an equilibrium rate of profit corresponding to a given ratio of capital to labour then cannot be applied without a great deal of complication. It is for this reason that the assumption of free competition (in the long-period sense) is necessary to it.

Given that competition establishes a uniform rate of profit throughout the economy, and given technical knowledge and the quantity of capital (in terms of product) there is one value of the wage rate which is compatible with full employment of any given labour force.

The neo-classical economists derived from this proposition a doctrine which cannot, in fact, be based on it. They maintained that the level of wages determines the amount of employment, and that, when unemployment occurs, workers (unless frustrated by the misguided policy of trade unions) offer themselves at a lower real wage rate than that ruling, and go on doing so till all are employed.

This doctrine was challenged by Keynes, on the ground that the wage bargain does not determine the real wage. Keynes' argument was developed to deal with short-period situations, but it applies with full force to equilibrium positions. A change in the peanut price, or the money price, of a man hour of labour alters the equilibrium price, in terms of peanuts or of money, of each commodity proportionally and leaves the equilibrium rate of profit and of wages unchanged. In short, the purchasing power, whether of money or of peanuts, over commodities in general, is governed by its purchasing power over labour, and a change in the peanut price or the money price of labour does not affect the price of labour in terms of commodities in general.

From the point of view of any one employer his wage bill in terms of his own product is by no means the same thing as the real wage which the worker gets. The wage bill to the capitalist and the wage received by the workers are the same only in a strictly one-commodity world, as Ricardo saw when he imagined a system in which corn is the only product, and the wage contract is made in terms of corn.¹ Wage bargaining is conducted in terms of what the worker gets, and the wage which enters into the wage bargain is not the same thing as the wage which determines the quantity of employment. Thus the conception of the level of employment being determined by the wage bargain cannot be expressed (outside the corn economy) in a way which has a meaning.²

¹ *Works and Correspondence of David Ricardo*, ed. Sraffa, Vol. I, p. xxxii.

² The difficulty cannot be evaded by postulating that wages are paid in kind without the use of money. To specify a "non-monetary" or "non-Keynesian" economy it is not sufficient to postulate that the society in question has not yet got round to inventing a standardised medium of exchange. We must postulate that the very idea of generalised purchasing power is unknown, so that each separate employer pays his workers in his own product, and the workers barter the products amongst themselves without any triangular dealing.

ACCUMULATION

What becomes of the neo-classical doctrine if we read it the other way round : that the rate of profit tends to be such as to permit all the capital that comes into existence to be employed ? Suppose that the wage rate has been established at a level which yields some conventional minimum real wage, and that, the technique having been chosen which maximises the rate of profit, the quantity of capital in existence does not employ all available labour, so that there is a reserve of unemployment. Accumulation can then proceed at a constant factor ratio and constant rate of profit until all available labour is employed. If population is increasing at least as fast as capital is accumulating, full employment is never attained, and the expansion of the economy can continue indefinitely (we have postulated that there is no scarcity of land, including all non-produced means of production).

So far the argument is dismally simple. What are we supposed to imagine to happen when there is full employment in the long-period sense, that is, when there is sufficient plant in existence to employ all available labour ? One line of argument is to suppose that the capitalists who are accumulating act in a blindly individualistic manner, so that a scramble for labour sets in ; the money (or peanut) wage rate is bid up, and prices rise in an indefinite spiral. (It is of no use to bring the financial mechanism into the argument, for if the supply of the medium of exchange is limited, the interest rate is driven up ; but what the situation requires is a fall in the rate of interest, to encourage the use of more mechanised techniques.)

Or we may postulate that the capitalists, while fully competitive in selling, observe a convention against bidding for labour—each confines himself to employing a certain share of the constant labour force. Then any one who wishes to increase the amount of capital that he operates shifts to a more mechanised technique. Those who first make the change may be supposed to compete for wider markets and so to reduce prices relatively to money wages. The Ricardesque effect is thus brought into play, and the switch to more mechanised techniques proceeds at a sufficient rate to absorb new capital as it accrues. Alternatively, we might imagine that an excessive number of plants of the less mechanised type are actually built, and that their redundancy, relatively to labour to man them, reduces profit margins, so that the wage rate rises and brings the Ricardesque effect into play. (Whichever line we follow the argument is necessarily highly artificial, for in reality the state of trade is the dominant influence on investment. The situation which promotes the mechanisation of production is full employment and full order books, that is to say, a scarcity of labour relatively to effective demand, but the equilibrium assumptions do not permit us to say anything about effective demand.)

Somehow or other, accumulation may be conceived to push down the rate of profit, and raise the factor ratio.

But the very notion of accumulation proceeding under equilibrium conditions at changing factor ratios bristles with difficulties. The rate at which the factor ratio rises is not governed in any simple way by the pace at which accumulation goes on—it depends upon the extent to which the rising wage rate causes capital to be absorbed

Several neo-classical economists have invented, as a prophylactic against unemployment, " commodity money " in various forms, which would produce an effect as though the wage contract were made in terms of the composite commodity representing all output. As a practical proposal, such a scheme is rather fanciful, but as a pedagogical device explaining the nature of money, it is excellent. Another way of imagining the same result is to suppose that the workers are paid in terms of a percentage of the money proceeds of the firm employing them, that is, that the workers take on the risks of entrepreneurship. This also starts an interesting train of thought.

Robinson Crusoe lived in a completely amonetary world, as Defoe makes him point out in his meditations over a parcel of coins. For this reason he is the patron saint of neo-classical economics.

by the Wicksell effect. Moreover, the effect of a given change in the factor ratio depends upon the speed at which it is made, relatively to the length of life of plant. If capital per man is rising rapidly some capitalists will still be operating Delta and Gamma plants while others have already installed Alpha plants. (In these conditions the trick assumptions become very tricky indeed.)

Even if we can find a way through these complications, there remains the formidable problem of how to treat expectations when the rate of profit is altering. An unforeseen fall in the rate of profit ruptures the conditions of equilibrium. Capitalists who are operating on borrowed funds can no longer earn the interest they have contracted to pay, and those operating their own capital find themselves in possession of a type of plant that they would not have built if they had known what the rate of profit was going to be.

On the other hand, if we postulate that accumulation goes on in the expectation of a gradually falling rate of profit, the whole basis of the analysis becomes immensely complicated. We can no longer argue in terms of a single interest rate. There is a complex of rates for loans of different lengths, the rates for shorter terms standing above the rates for longer terms. Moreover, the pace at which the rate of profit falls as the factor ratio rises is dictated by technical conditions. Over its early reaches the factor-ratio curve may be supposed to be steep, with the rate of profit falling slowly. Then it passes over a hump, with a rapid fall in the rate of profit, and flattens out again with a lower but more slowly falling rate of profit. To be correct, the expectations of the capitalists cannot merely be based on past experience but require a highly sophisticated degree of foresight.

Thus the assumptions of equilibrium become entangled in self-contradictions if they are applied to the problem of accumulation going on through time with a changing factor ratio. To discuss accumulation we must look through the eyes of the man of deeds, taking decisions about the future, while to account for what has been accumulated we must look back over the accidents of past history. The two points of view meet only in the who's who of goods in existence to-day, which is never in an equilibrium relationship with the situation that obtains to-day.

In short, the comparison between equilibrium positions with different factor ratios cannot be used to analyse changes in the factor ratio taking place through time, and it is impossible to discuss changes (as opposed to differences) in neo-classical terms.

The production function, it seems, has a very limited relevance to actual problems, and after all these labours we can add little to the platitudes with which we began : in country Gamma, where the road builders use wooden shovels, if more capital had been accumulated in the past, relatively to labour available for employment, the level of real wages would probably have been higher and the technique of production more mechanised, and, given the amount of capital accumulated, the more mechanised the technique of production, the smaller the amount of employment would have been.

A CHANGE IN TECHNICAL KNOWLEDGE

It remains to inquire whether the analysis can be applied to a change in our third basic datum, the state of technical knowledge.

In the neo-classical system, technical knowledge is altered by inventions. An "invention" is conceived not merely as an isolated innovation in a method of production or the design of equipment. It is a discovery which has a wide range of applications and which raises the productivity of labour over a wide range of factor ratios.

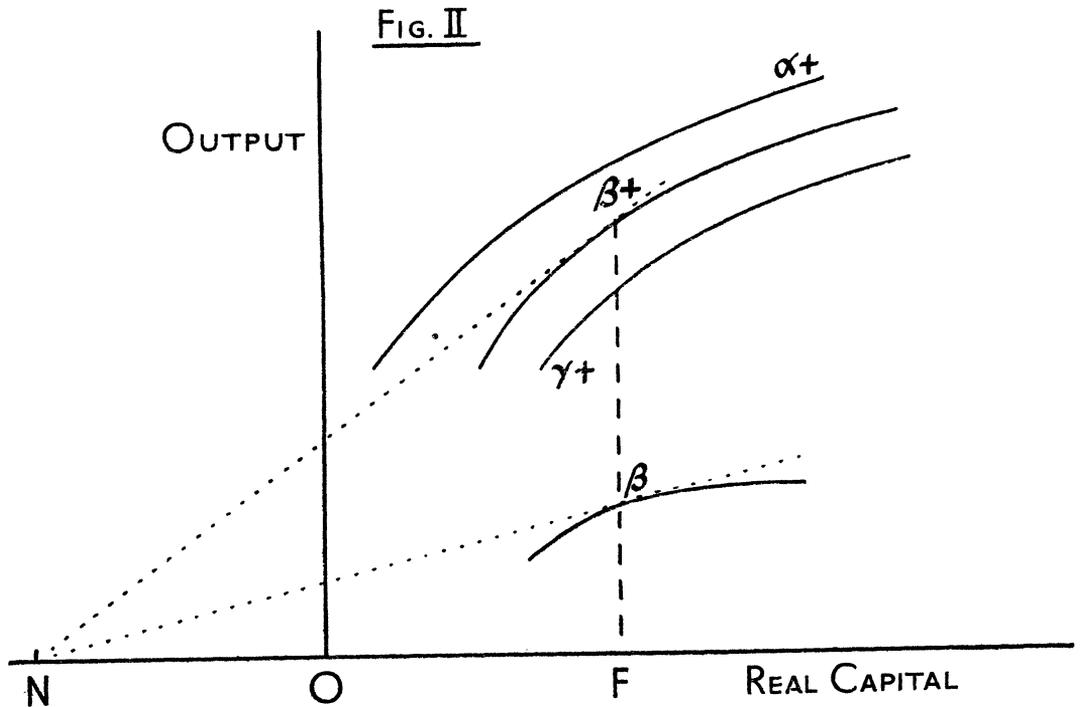
The argument is usually conducted in terms of the effect of inventions upon the output and the relative earnings of given quantities of capital and labour. Thus we

are supposed to imagine an economy taking a standing jump from one stationary position to another and landing itself in equilibrium in a new state of knowledge with the same supplies of factors of production as before. This makes the contradictions involved in the neo-classical conception of equilibrium and the ambiguities involved in the conception of a constant quantity of capital more formidable than ever.

However, it is possible once again to distil something from the ingredients of the argument. Technical progress may be incorporated into the conditions of equilibrium if we postulate that inventions are expected to be made at a certain rate and that, in fact, they do succeed each other in a smooth and regular manner. All capital goods are, therefore, provided with obsolescence funds, which allow equipments to be transmogrified to suit new techniques without any loss of capital in terms of product. An invention then does not give a shock to the system rupturing the conditions of equilibrium, and we can continue to rule out problems of effective demand by assuming that saving is geared to investing so that if accumulation is taking place it goes on at a steady rate without any perturbations.

We are also obliged to put a heavy weight on the trick assumptions, for it is impossible to compare the efficiency of different methods of production unless they are designed for similar outputs.

Subject to these provisos, we can (with due reservations) make use of the foregoing analysis to compare equilibrium positions of the economy at two points of time. In the first position, say, Beta technique is in use.



In the diagram β is the point of equilibrium on the factor-ratio curve in the first position ; the factor ratio is OF ; and the elasticity of the productivity curve through β is e (the productivity curve is drawn up using the Beta rate of profit as the notional

interest rate : its elasticity is equal to the share of profit in output when equilibrium obtains at β).

Now draw the productivity curve for the new state of knowledge using the rate of profit which obtains at β as the notional interest rate. Let $\beta+$ be the point on this curve corresponding to the factor ratio OF .

If the elasticity of the curve at $\beta+$ were equal to e , the wage rate at factor ratio OF would have been raised by the improvement in technique in the same proportion as output. Capital per unit of output, and the rate of profit on capital would, therefore, be the same as in the first position. If the elasticity at $\beta+$ were less than e , the wage rate would be raised more than in proportion to output, and the share of profit in output and the rate of profit on capital would be reduced (readjusting for a lower notional interest rate reinforces these relations). The converse would hold if the elasticity at $\beta+$ were greater than e .

A lower elasticity of the productivity curve implies that as we move up the new hierarchy of techniques, Gamma + to Beta + to Alpha +, output per man rises proportionately less than was the case with the old hierarchy. There is, so to say, less scope in the new situation for the Ricardesque effect to work. Thus inventions which alter the shape of the productivity curve in this way (over the relevant range of factor ratios) may be called *unfavourable* to capital, those which leave the elasticity at a given factor ratio unchanged, *neutral*, and those which raise it, *favourable* to capital.

The nature of the technical change does not by itself determine a new position of equilibrium. That depends also upon how much accumulation took place while the technical change was being made. As we have seen, for the factor ratio to remain unchanged, when the inventions have been neutral, capital must have increased in the same proportion as output. With any smaller amount of accumulation the factor ratio is reduced, the system must be supposed to have moved in the direction of, say, $\gamma+$ or $\delta+$, and the share of profit in output and the rate on capital are higher than they were at β .

Thus the effect of inventions, unless they are highly unfavourable to capital in a technical sense, is to raise the rate of profit obtainable by a given amount of capital, and to increase its relative share in output.

The above analysis throws some light on the controversy between the economists and the Luddites, and supports Ricardo's argument *On Machinery*.¹ When the nature of inventions is sufficiently favourable to capital, the wage rate at the factor ratio OF is reduced by them. But if the wage rate ruling at β provided a real wage no greater than the subsistence minimum, it is impossible for the wage rate in a new position of equilibrium to be any lower. There is a possible position of equilibrium at a higher factor ratio, say at $\alpha+$, where the wage rate is the same as that which prevailed at β . But to provide a higher factor ratio for a constant labour force at a constant wage rate requires an increase in the quantity of capital. If the quantity of capital has not increased sufficiently, the higher factor ratio can be attained only by reducing employment. There is then a "substitution of capital for labour" in a literal and brutal sense.

To return to our road builders—if inventions made it profitable to introduce bulldozers in country Gamma the workers there would be much worse off than they were with wooden shovels, unless at the same time there were a sufficient accumulation of capital to provide the labour displaced from road building with employment in other industries.

This scheme of analysis provides the basis for the model of a capitalist system

¹ *Principles*, Third Edition, Chapter XXXI.

enjoying continuous expansion. Where technical progress is neutral and accumulation goes on at just such a rate as to keep the ratio of capital to output constant, the share of capital and labour in output and the rate of profit on capital remain constant and the model is free from "internal contradictions."¹

In other cases the complications arising from changes in the distribution of income between classes and changing expectations of profits are too great to be digested by the assumptions of equilibrium, and it is idle to pursue the argument any further without taking account of the problem of effective demand.

CONCLUSION

The tenor of our argument has been mainly negative and the level of abstraction maintained in the analysis is very high. Nevertheless, we can draw some general conclusions from it. The conclusions sound obvious enough, but perhaps that is all to the good, as it shows that the propositions drawn from the abstract argument are not in conflict with common sense.

The rate of profit on capital will tend to be higher, and real wages lower :

(1) the more plentiful are the technical opportunities for mechanising production ;

(2) the slower is the rate of capital accumulation in relation to the growth of population ;

(3) the weaker is the force of competition (and the weaker is the bargaining power of the workers, when competition is weak).

Given the degree of competition and the rate of growth of population, the course of the rate of profit over the long run (abstracting from short-period fluctuations) depends on the interaction between technical progress and the rate of accumulation. Technical discoveries (unless extremely unfavourable to capital) are continuously tending to raise the rate of profit and accumulation is tending to depress it. Prosperous capitalist economies are those where the rate of profit is falling in spite of rapid technical progress, and miserable ones those where the rate of profit is high in spite of technical stagnation.

Cambridge.

JOAN ROBINSON.

APPENDIX

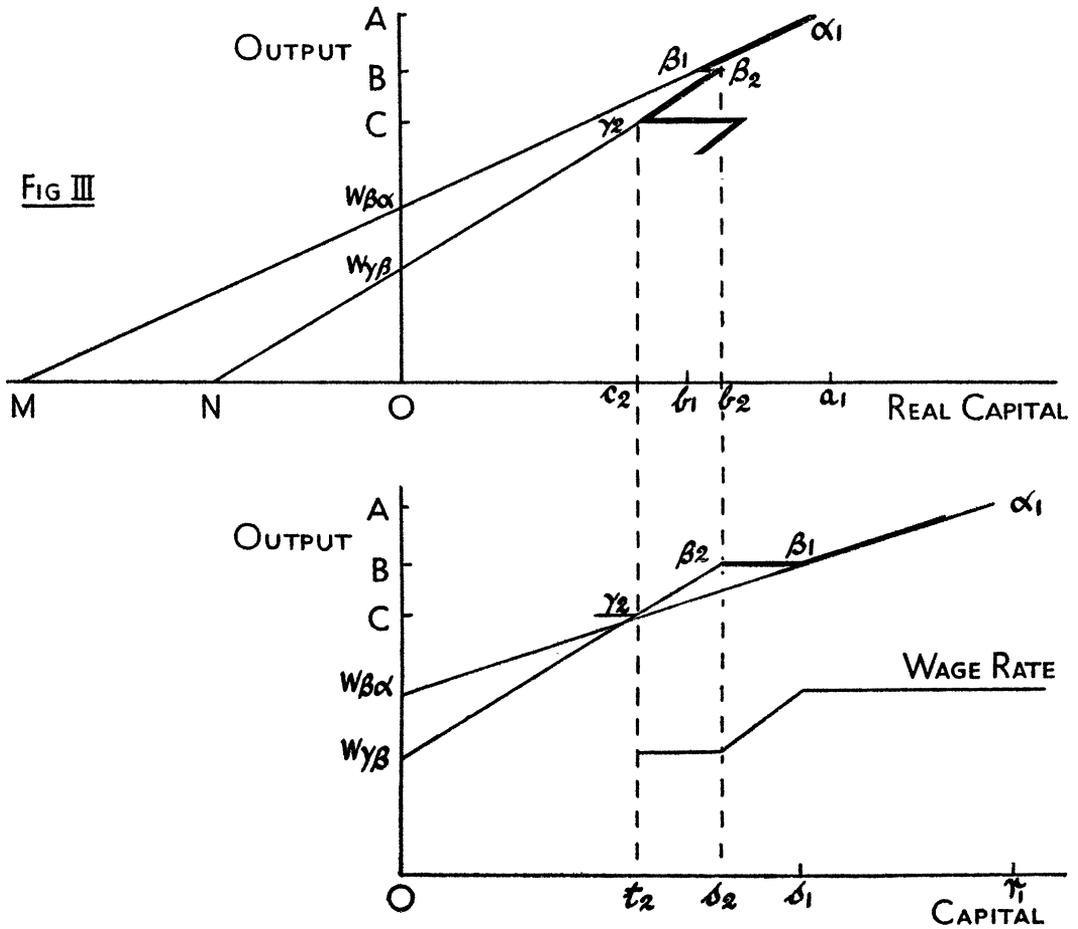
THE FACTOR-RATIO CURVE AND THE CAPITAL-RATIO CURVE

The relations between the factor ratio and the ratio of capital (in terms of product) to labour employed discussed above (p. 93) can be set out diagrammatically as follows :

The upper half of Fig. III (overleaf) repeats Fig. I. The slope of the straight line, $\gamma_2 \beta_2$ is the ratio of an increment of output to the increment of real capital with which it is associated, and at a constant wage rate an increment of output is an increment of profit ; thus this slope is the ratio of the increment of profit to an increment of real capital. Therefore, by producing $\beta_2 \gamma_2$ to cut the y axis in $w_{\gamma\beta}$ we find the wage rate, $OW_{\gamma\beta}$, at which Gamma and Beta techniques are indifferent. When output is OC , $W_{\gamma\beta}C$ is profit per man employed, and when output is OB , $W_{\gamma\beta}B$ is profit per man. Similarly, $OW_{\beta\alpha}$ (when $\alpha_1\beta_1$ cuts the y axis in $W_{\beta\alpha}$) is the wage rate at which Beta and Alpha techniques are equally profitable.

¹ At this point the argument joins on to that set out in *The Rate of Interest and Other Essays*, p. 90.

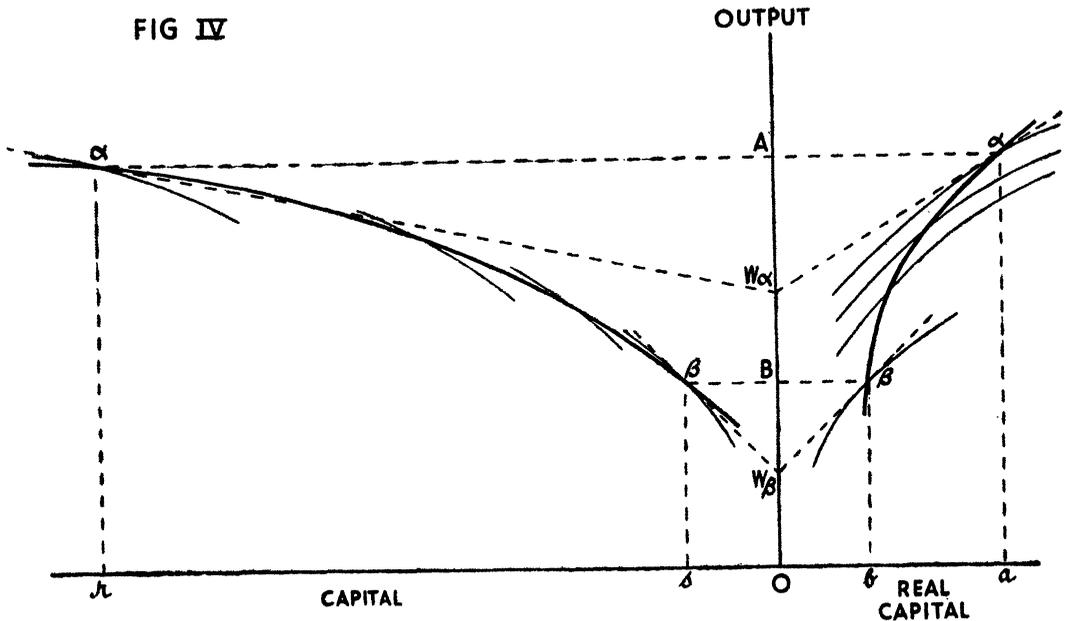
At any factor ratio, the amount of capital per man (that is, capital in terms of product) is equal to real capital per man multiplied by the wage rate. Thus, at β_2 , capital per man is $Ob_2 \cdot OW_{\gamma\beta}$ and the rate of profit on capital ruling over the range $\gamma_2\beta_2$ is equal to $W_{\gamma\beta}B/Ob_2 \cdot OW_{\gamma\beta}$. Produce $\beta_2\gamma_2$ to cut the x axis in N . Then $OW_{\gamma\beta}/ON = W_{\gamma\beta}B/Ob_2$. Therefore, the rate of profit on capital is equal to $1/ON$. Similarly the rate of profit over the range $\beta_1\alpha_1$ is $1/OM$ when $\alpha_1\beta_1$ cuts the x axis in M .



We can now draw a curve relating output to the ratio of capital to labour employed, that is, to capital per man in terms of product. In the lower half of Fig. III the y axis is identical with that of the upper half; the x axis measures capital per man, instead of real capital per man. Physical capital, that is the who's who of capital goods corresponding to each rate of output, is unaffected by the manner in which the items composing it are valued, and points given the same names (β_1 , a_2 , etc.) correspond to identical technical situations. The distance Os_2 on the lower x axis is equal to the area $Ob_2OW_{\gamma\beta}$ in the upper half of the diagram. To facilitate comparison, we take the distance $OW_{\gamma\beta}$ as the unit for x on the lower axis, and make the distance Ob_2 equal to Os_2 . Then the two straight lines $\gamma_2\beta_2$ are identical.

At β_2 all men are employed with Beta technique, and a rise in the factor ratio and the capital ratio requires a rise in the wage rate to $OW_{\beta\alpha}$. Capital per man then increases to Os_1 on the lower x axis, while real capital per man falls to Ob_1 on the upper x axis. (In this case the rise in the wage rate more than offsets the fall in real capital due to a lower rate of profit. The Wicksell effect outweighs the interest effect.)

The slope of $\gamma_2\beta_2$ in the lower half of the diagram ($W_{\beta\gamma}B/Os_2$) is the rate of profit on capital which obtains when Gamma and Beta techniques are indifferent, and the slope of $\beta_1\alpha_1$ is the rate of profit when Beta and Alpha techniques are indifferent. The greater length of s_1r_1 on the lower x axis compared to b_1a_1 and the smaller slope of the corresponding straight line $\beta_1\alpha_1$ reflects the excess of the $\beta\alpha$ wage rate over the $\gamma\beta$ wage rate.



Smoothing out discontinuities in the productivity curves, we may draw a pair of diagrams for the factor-ratio curve and the capital ratio curve as in Fig. IV. We take real capital as the x axis in the right hand part of the diagram and capital in the left hand.

Corresponding to each of the family of productivity curves in the right hand is what we may call a pseudo-productivity curve on the left hand, showing what capital per man would be if the wage rate were that which is compatible with the rate of interest used in drawing the corresponding productivity curve. Each pseudo productivity curve has a meaning only in the neighbourhood of the point of equilibrium corresponding to the wage rate on the basis of which it is drawn.

The factor-ratio curve cuts the family of productivity curves from below as it rises. The capital-ratio curve cuts the pseudo productivity curves from above.

The point β on each curve corresponds to the output OB . Draw a tangent to the productivity curve through β . By the same reasoning as above, its intercept on the y axis, OW_{β} , is the wage rate at which the corresponding technique will be in use. As before, we take OW_{β} as the unit for the capital axis, so that the tangent to the pseudo-

productivity curve through β on the capital-ratio curve is drawn as identical with that to the productivity curve through β on the factor-ratio curve. The elasticity of the tangent, $W_{\beta}B/OB$, is the ratio of profit to output, or relative share of capital in product.

Similarly, draw tangents at a . The elasticity of the pair of tangents is the same, the greater distance to the left of the left hand position of a compensating for the smaller slope of the tangent. The slope of the tangent on the left (W_aA/Aa) is the rate of profit on capital.

It is convenient for some purposes to conceive the productivity curves as continuous, each technique requiring an indefinitely small increase in capital per man compared to the last. But it is hard to picture what this would mean in reality (even in the famous wine cellar). Moreover, the essential nature of the mechanism of the relationship between the rate of wages and the choice of technique can be understood only in terms of discontinuous curves; we must, therefore, think of a point, such as β or a in Fig. IV, as being a small straight line segment of a curve, over which two techniques are indifferent, rather than as representing the product of a single technique.

The geometry reveals a curious possibility.¹ It may happen that, over a certain range, a reduction in the rate of interest produces a larger reduction in the capital cost of the equipment for a lower than for a higher technique, so that successive wage tangents become steeper as the rate of profit falls. They may then find contact with productivity curves at successively lower points, so that a lower rate of profit (and a higher wage rate) results from the use of a *less* mechanised technique. This might occur if the plant required for less mechanised techniques had a much longer gestation period or working life, so as to be much more sensitive to the interest rate than that for more mechanised techniques. This "perverse" behaviour of the factor-ratio curve, where it occurs at all, can be only over a certain range. At very low values of the rate of interest the differential effect (as between techniques) of changes in the interest rate must be small, so that there must be an upper range, on the way to Bliss, over which the factor ratio curve rises to the right as the rate of interest falls; and the degree of mechanisation must have reached a certain level before there is any scope for it to fall, so that there must be a lower range over which the factor ratio curve rises. In the case where the curve at first rises in a "normal" manner, then falls to the left "perversely," then rises again, there is evidently a certain range of techniques which provide possible positions of equilibrium at three different wage rates. A good deal of exploration of the possible magnitude and behaviour of the interest effect is needed before we can say whether the above is a mere theoretical rigmarole, or whether there is likely to be anything in reality corresponding to it.

¹ This was pointed out to me by Miss Ruth Cohen.

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[Footnotes]

² **Knut Wicksell: A Centennial Evaluation**

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